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SUPPLEMENT TO REPORT NO. NADC-72185-AD → (907731) 7300185
VIBRATION ENERGY SUMMATIONS IN RESPONSE
DOMAIN FOR COULOMB DAMPED ELASTIC
SYSTEMS WITH SINUSOIDAL EXCITATION

AIRTASK NO. WF53-532-404
WORK UNIT NO. 1

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DEPARTMENT OF THE NAVY
NAVAL AIR DEVELOPMENT CENTER

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DEPARTMENT OF THE NAVY
NAVAL AIR DEVELOPMENT CENTER
WARMINSTER, PA. 18974

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REPORT NO. NADC-73153-81 20 December 1973

SUPPLEMENT TO REPORT NO. NADC-72185-AD VIBRATION ENERGY SUMMATIONS IN RESPONSE DOMAIN FOR COULOMB DAMPED ELASTIC SYSTEMS WITH SINUSOIDAL EXCITATION

AIRTASK NO. WF53-532-404
Work Unit No. 1

This report supplements the previous phase report covering response energy summation analysis for a rigidly-connected coulomb damped elastic structure model, when subjected to all controlled patterns of sinusoidal vibration. Specimen curve evaluations for extended parameter ranges are facilitated by the use of computer plotting programs.

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F O R W A R D

This report is a supplement to Report No. NADC-72185-AD, Vibration Energy Summations in Response Domain for Coulomb Damped Elastic Systems with Sinusoidal Excitation, of 2 February 1973.

Sections IX through XIV of Report No. NADC-72185-AD are devoted to the evaluation of the E''_x expressions applied to the revised sinusoidal specimen curve. The results are displayed in tables III through XXVII and in figures 3 through 28 of that report.

Since the publication of Report No. NADC-72185-AD, a computer plotting program has been developed for the functions E''_{s1} through E''_{s12}. Apart from greatly simplifying the evaluations, the plotting program affords several additional advantages, which will be apparent upon examination.

This report presents the computer plotting program, together with 30 plots which completely display E''_{s1} through E''_{s12}. No applicable discussion or philosophy will be repeated herein unless inadequately treated in Report No. NADC-72185-AD. The reader is therefore referred to that report as the required basis for this supplement.

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DEVELOPMENT OF COMPUTER PLOTTING PROGRAM

Examination of the 10 energy summation functions for swept excitation was made to determine the type and capacity of computer to be used, together with the input/output formats which would best facilitate the evaluations. The following observations were made:

1. All elements of the functions are expressed in alphanumeric and transcendental symbology.

2. Apart from their arithmetic coefficients, the two most complex terms, i.e., $\ln(W-1) \pm \ln(W+1)$ and $1/(W^2-1)$ or $W/(W^2-1)$, were common to all summation functions.

3. The most useful display format, for illustration and comparison as well as direct application, is a set of curve families, one for each summation function. Tabular printout is not considered to be of substantial additional value.

The decision was therefore made to abandon further consideration of both computer group services and direct on-line remote terminal access to an available CDC 6600 facility. Instead, the Hewlett-Packard 9100 B programmable calculator was chosen, together with its companion, the 9125 plotter. This calculator provides direct keyboard entry in mathematical terms, requiring neither the knowledge of special language nor the use of language software interfaces. Further, it offers adequate storage and stacked subroutine capability.

After many trials, a modular plotting program was developed which uses two subroutines, one for the calculation of $E''_x(W)/K_x$ for a single value of W, and the other for the plotting and iteration of W.

In section VII of Report No. NADC-72185-AD, 10 letter coefficients were chosen, and the following generalized summation expressions were written:

For logarithmic sweep with $W \geq \sqrt{d}$,

$$\frac{E''_x(W)}{K_x} = \left[M_x \ln\left(\frac{W-1}{W+1}\right) + O_x\left(\frac{2W}{W^2-1}\right) + \frac{B_x}{W} + \frac{D_x}{W^3} + \frac{L_x}{W^5}\right].$$

For linear sweep with $W \geq \sqrt{d}$,

$$\frac{E''_x(W)}{K_x} = \left[N_x \ln(W^2-1) + P_x\left(\frac{2}{W^2-1}\right) + A_x \ln(W) + \frac{C_x}{W^2} + \frac{J_x}{W^4}\right].$$

Further, in section VI of Report No. NADC-72185-AD, it was shown that

$$\frac{E''_{s2}(W)}{K_{s2}} = \frac{E''_{s5}(W)}{K_{s5}}, \text{ and } \frac{E''_{s8}(W)}{K_{s8}} = \frac{E''_{s11}(W)}{K_{s11}}.$$

Therefore, only 8 expressions for $E''_x(W)/K_x$ suffice for the 10 summation functions. The Σ'' subroutine has "modules", simple columnar step entries, for the calculation of the coefficients, the functions of W, and the (+) and (-) accumulation of the B_x/W and $A_x \ln(X)$ terms, respectively. For those functions in which one or more of the generalized terms do not appear, the corresponding modular step columns yield zero for their coefficients. The PLOT/ΔW subroutine contains modules for X and Y scaling, W correction for Y scaling, ΔX plot and K^2. The main program contains modules for $(KW)^2$max, K^2, W^2min, d^2min, R^2, d^2max, Σ_I (for $W_1 \le W \le W_L$), and Σ_{III} (for $W_L \le W \le KW_1$).

The operation of the plotting program is quite simple. However, a few explanations will serve to clarify the main block diagram, figure 1. In section IX of Report No. NADC-72185-AD, the three cases were introduced, which are summarized below:

<u>Case I (locked condition):</u> $(W_L > KW)$

$$E''_x(W)\Big]_W^{KW} = E_x(W)\Big]_W^{KW}$$

<u>Case II (relative motion across damper):</u> $(W_L < W)$

$$E''_x(W)\Big]_W^{KW} = K_x \int_W^{KW} Q_x(W)\,dW$$

<u>Case III (locked damper breaks loose during sweep):</u> $(W_1 < W_L < KW_1)$

$$E''_x(W)\Big]_W^{KW} = E_x(W)\Big]_W^{W_L} + K_x \int_{W_L}^{KW} Q_x(W)\,dW$$

The computer program compares W_L first with W_1, then with KW_1. At the first junction, if $(W_L < W)$, the FLAG is set, and the Σ'' subroutine calculates $[\Sigma''(KW) - \Sigma''(W)]$, indicating a pure Case II condition. At the second junction, since the FLAG was set, $[E''(KW) - E''(W)]$ is plotted,

and $(KW)^2$ is incremented (decreased). Eventually, the first junction will find ($W_L \nless W$), i.e., ($W < W_L < KW$). At this point, the FLAG is no longer set, and the Σ'' subroutine calculates $[\Sigma''(KW) - \Sigma''(W_L)]$, indicating a mixed, or Case III, condition. At the second junction, since the FLAG was not set, $[\Sigma''(KW) - \Sigma''(W_L)]$ is not plotted. The Σ'' subroutine has already stored it. Instead, $[\Sigma(W_L) - \Sigma(W)]$ is calculated, then added to $[\Sigma''(KW) - \Sigma''(W_L)]$. Finally, $[E''(KW) - E''(W_L) + E(W_L) - E(W)]$ is plotted, and $(KW)^2$ is incremented (decreased). Up to this point, the third junction has found ($W_L < KW$). However, the third junction will eventually find ($W_L > KW$), as W is decreased still further. At this point, $[\Sigma(KW) - \Sigma(W)]$ is calculated, indicating a pure Case I condition. $[E(KW) - E(W)]$ is plotted and the incremented $(KW)^2$ is re-entered below the third junction. The Case I summation plotting continues until the fourth junction finds ($W < W_{min}$). The rest of the main program should require no explanation. Figure 2 shows the PLOT/ΔW subroutine block diagram, and is presented to clarify its single junction, which provides for pen lift whenever Yplot exceeds the maximum ordinate of the paper.

It should be noted that although the main program is written in terms of W, KW and W_L, the Σ subroutine is written in terms of W_1 and W_2. This is done for subroutine program clarity. The upper limit W_2 is always KW, while the lower limit W_1 is either W for a FLAG pass (Case II) or W_L for a non-FLAG pass (Case III).

The following symbols are used in the program, and supplement those listed in Report No. NADC-72185-AD.

d_{min} d_{max} - Iterative program damping parameter limits

R - The incrementing ratio for d

Σ_I - E_x/K_x for Case I

Σ_{III} - E_x/K_x for Case III

Σ_P - General symbol used in the plotting subroutine. It represents Σ_I (Case I), Σ'' (Case II), or $\Sigma''+\Sigma_{III}$ (Case III).

Σ''_{MN} - $M_x \ln(\frac{W-1}{W+1})$ or $N_x \ln(W^2-1)$

Σ''_{OP} - $O_x \frac{2W}{W^2-1}$ or $P_x \frac{2}{W^2-1}$

Σ''_{BA} - B_x/W or $A_x \ln(W)$

Σ''_{DC} - D_x/W^3 or C_x/W^2

Σ''_{LJ} - L_x/W^5 or J_x/W^4

Σ'' - $(\Sigma''_{MN} + \Sigma''_{OP} \pm \Sigma''_{BA} + \Sigma''_{DC} + \Sigma''_{LJ})$

$(KW)^2{}_{max}$, $W^2{}_{min}$ - Iterative program W limits

X_{scale}, Y_{scale} - Plotting subroutine constant entries which yield suitable horizontal and vertical scaling rates

X_{plot}, Y_{plot} - The numerical values actually fed to the plotter servos

The complete program is presented on the following pages. Figure 1 outlines the overall program in block diagram form. Figure 2 is a block diagram of the PLOT/ΔW subroutine. Figure 3 is a block diagram of the Σ" subroutine.

Tables I through XV, which follow the program, are self-explanatory. Reference is made to tables I through VI in the program, to table II and tables VII through X in the PLOT/ΔW subroutine, and to tables XI through XV in the Σ" subroutine. It will be noted that three modules are not tabulated; these entries are arbitrary, not mandatory.

EVALUATION OF RESPONSE ENERGY SUMMATIONS FOR SINUSOIDAL SPECIMEN CURVE

Using the computer plotting program presented in the preceding section, and entering the appropriate adjustment and scaling for each of the 10 specific swept sinusoidal energy summations expressions, 30 curve families have been plotted. Three of these plots present the summations for each specific expression. Figures 4 through 33 present the 10 curve family groups as plotted on the Hewlett-Packard Model 9125 plotter, for the sinusoidal specimen curve.

The overall damping parameter range used in Report No. NADC-72185-AD is $1.0186 \leq d \leq 12.5$. The corresponding ranges of η and W_L are $0.80 \leq \eta \leq 9.818$, and $1.0092 \leq W_L \leq 3.535$.

Using the computer plotting program, the above ranges have been expanded, particularly in the low damping domain. The corresponding ranges follow:

$1.000001 \leq d \leq 16.0$, $0.7854 \leq \eta \leq 12.57$, and $1.0000005 \leq W_L \leq 4.0$.

The overall natural frequency range has a constant lower limit in each range. The upper limit has been determined for each plot such that the lowest value of fn for Case I (locked damper) is shown. The resulting summation value is therefore constant for all higher values of fn.

For all three ranges, the following two families of damping parameter values have been used:

<u>Family 1:</u> d = 1.000001, 1.000002, 1.000005, 1.00001, 1.00002, 1.00005, 1.0001, 1.0002, 1.0005, 1.001, 1.002, 1.005 and 1.01.

<u>Family 2:</u> d = 1.01, 1.02, 1.05, 1.1, 1.2, 1.5, 2.0, $2\sqrt{2}$, 4.0, $4\sqrt{2}$, 8.0, $8\sqrt{2}$ and 16.0.

The family assignments, fn ranges, and scaling data, for figures 4 through 33, have been tabulated in table XVI for ranges I and II, and table XVII for range III.

6/13/73

Title $E''_{S1}, E''_{S2}, E''_{S3}, E''_{S5}, E''_{S6}, E''_{S7}, E''_{S8}, E''_{S9}, E''_{S11}$ and E''_{S12}

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Step	Key	Code	Display x	Display y	Display z
00	CLEAR		0	0	0
01	d^2_{min}				
02					
03					
04					
05			d^2_{min}		
06	X→()				
07	d		$d^2 = W_L^4$		
08	Table I				
09					
0a					
0b			$(KW)^2_{max}$		
0c	X→()				
0d	f		$(KW)^2$		
10	↑			$(KW)^2$	
11	Table II				
12			K^2		
13	÷			W^2	
14	d		W_L^4		
15	√x		W_L^2		
16	(x≥y)?				
17	2				
18	1				
19	Y→()				
1a	c			W^2	
1b	SET FLAG				
1c	Go To				
1d	2				
20	5				
21	d		W_L^4		
22	√x		W_L^2		
23	X→()				
24	c		W_L^2		
25	Go To				
26	Sub/Ret				
27	−				
28	3			Σ''	
29	6				
2a	FLAG			Σ''	
2b	4	Yes			
2c	b				
2d	f		$(KW)^2$		

Step	Key	Code	Display x	Display y	Display z
30	↑		$(KW)^2$	$(KW)^2$	Σ''
31	Table II				
32			K^2		
33	÷			W^2	
34	Y→()				
35	c			W^2	
36	d		W_L^4		
37	√x		W_L^2		
38					
39					
3a					
3b					
3c					
3d	Table III				
40					
41					
42					
43					
44					
45					
46					
47					
48				Σ'_{III}	
49	e		Σ''		
4a	+		$(\Sigma''+\Sigma'_{III})$		
4b	Go To				
4c	Sub/Ret		PLOT ΔW		
4d	a				
50	3				
51	d		W_L^4	$(KW-\Delta W)^2$	0
52	√x		W_L^2		
53	(x>y)?				
54	5	Yes			
55	a				
56	↓	No	$(KW-\Delta W)^2$	0	
57	Go To				
58	0				
59	c				
5a	Table II			$(KW-\Delta W)^2$	0
5b			K^2		
5c	÷			W^2	
5d	Y→()				

Step	Key	Code	Display x	Display y	Display z
60	c			W^2	0
61					
62					
63					
64					
65					
66	Table IV				
67					
68					
69					
6a					
6b					
6c					
6d				Σ_I	
70	Go To		PLOT ΔW		
71	Sub/Ret				
72	a				
73	3				
74	Table II			$(KW-\Delta W)^2$	0
75			K^2		
76	÷			W^2	
77					
78					
79	Table V				
7a					
7b					
7c					
7d					

Storage	
f	$(KW)^2$
e	Σ''
d	d^2
c	W^2, W_L^2, W^2
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Title

Step	Key	Code	Display x	Display y	Display z
80				W^2	0
1			W^2_{min}		
2	$(x \geq y)$?				
3	8	Yes			
4	9				
5 No	Go To				
6	5				
7	d				
8	d		d^2		
9	↑			d^2	W^2
a					
b	d^2_{max}				
c					
d			d^2_{max}		
90	$(x = y)$?				
1	a	Yes			
2	2				
3 No					
4	Table				
5	VI				
6			R^2		
7	×			$(Rd)^2$	
8	0		0		
9	↑			0	$(Rd)^2$
a	FMT				
b	↑				
c	Roll ↑		$(Rd)^2$		0
d	Go To				
a0	0				
1	6				
2	STOP				
3	c		W^2	Σ_p	
4					
5	Table				
6	VII				
7					
8			W_{corr}		
9					
a	Table				
b	VIII				
c	or				
d	IX				

Step	Key	Code	Display x	Display y	Display z
60					
1			Y_{scale}		
2	×			Y_{plot}	
3					
4					
5	Table				
6					
7	X				
8					
9			X_{scale}		
a	Go To				
b	–				
c	0				
d	0				
-00	↑			X_{scale}	Y_{plot}
1	c		W^2		
2	$\sqrt{x}$		W		
3	÷			X_{plot}	
4	3		3		
5	5		35		
6	0		350		
7	0		3500		
8	Roll ↑		Y_{plot}	3500	X_{plot}
9	$x \rightleftarrows y$		3500	Y_{plot}	
a	$(x \geq y)$?				
b	1	Yes			
c	6				
d No	Roll ↑		X_{plot}	3500	Y_{plot}
-10	FMT				
1	↑				
2	$x \rightleftarrows y$		3500	X_{plot}	
3	Go To				
4	1				
5	d				
6	d		d^2	Y_{plot}	X_{plot}
7	$\sqrt{x}$		d		
8	Roll ↓		Y_{plot}	X_{plot}	d
9	$x \rightleftarrows y$		X_{plot}	Y_{plot}	
a	FMT				
b	↓				
c	↑			X_{plot}	Y_{plot}
d					

Step	Key	Code	Display x	Display y	Display z
-20	ΔX_{plot}			X_{plot}	Y_{plot}
1			ΔX_{plot}		
2	+			$(X+\Delta X)_{plot}$	
3					
4					
5	Table				
6					
7	X				
8					
9			X_{scale}		
a	$x \rightleftarrows y$		$(X+\Delta X)_{plot}$	X_{scale}	
b	÷			$(W-\Delta W)$	
c	0		0		
d	$x \rightleftarrows y$		$(W-\Delta W)$	0	
-30	↑			$(W-\Delta W)$	0
1	×			$(W-\Delta W)^2$	
2	Table				
3	II		K^2		
4	×			$K(W-\Delta W)^2$	
5	Sub Rot				
6	f		W_2^2		
7	$\sqrt{x}$		W_2		
8	↑			W_2	W_2
9	↑				
a	1		1		
b	–			(W_2-1)	
c	Roll ↑		W_2	1	(W_2-1)
d	+			(W_2+1)	

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

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Title

Step	Key	Code	Display x	Display y	Display z
40	↓		(W_2+1)	(W_1-1)	(W_2-1)
1	$\boxed{\div}\ \boxed{\times}$			$(\div)$	
2	c		W_1^2		
3	$\sqrt{x}$		W_1		
4	↑			W_1	$(\div)$
5	1		1		
6	−			(W_1-1)	
7	↓		(W_1-1)	$(\div)$	
8	÷			$(\div)$	
9	c	Table XI	W_1^2		
a	$\sqrt{x}$		W_1		
b	↑			W_1	÷
c	1		1		
d	+			(W_1+1)	
50	↓		(W_1+1)	$(\div)$	
1	$\boxed{\times}\ \boxed{\div}$			$(\times)$	
2	|Y|			|X|	
3	↓		|X|	$(\div)$	
4	ln(x)		ln()		
5	↑			ln()	
6	d		d^2		
7	↑			d^2	ln()
8	5		5		
9	×			$5d^2$	
a	1		1		
b	−			$(5d^2-1)$	
c	4		4		
d	÷			$(\div)$	
60	↓		$(\div)$	ln()	
1	×			Σ''_{NM}	
2	Y→()				
3	e			Σ''_{NM}	
4	$\boxed{f}\ \boxed{//}$		W_2^2		
5	$\sqrt{x}$		W_2		
6	↑			W_2	Σ''_{NM}
7	f	Table XII	W_2^2		
8	↑			W_2^2	W_2
9	1		1		
a	−			(W_2^2-1)	
b	↓		(W_2^2-1)	W_2	
c	÷			$(\div)$	
d	c		W_1^2		

Step	Key	Code	Display x	Display y	Display z
70	↑		W_1^2	W_1^2	$(\div)$
1	1		1		
2	−			(W_1^2-1)	
3	$\boxed{c}\ \boxed{//}$		W_1^2		
4	$\sqrt{x}$		W_1		
5	X⇄Y		(W_1^2-1)	W_1	
6	÷			$(\div)$	
7	↓	Table XIII	$(\div)$	$(\div)$	
8	−			$(-)$	
9	2		2		
a	÷			$(\div)$	
b	d		d^2		
c	↑			d^2	$(\div)$
d	1		1		
80	−			(d^2-1)	
1	↓		(d^2-1)	$(\div)$	
2	×	Table XIII		Σ''_{102}	
3	0		0		
4	Acc+				
5	c		W_1^2		
6	$\sqrt{x}$		W_1		
7	$\boxed{\uparrow}\ \boxed{\ln(x)}$			W_1	Σ''_{102}
8	↑				W_1
9	f		W_2^2		
a	$\sqrt{x}$		W_2		
b	$\boxed{-}\ \boxed{\ln(x)}$			(W_1-W_2)	
c	$\boxed{R\downarrow}\ \boxed{x⇄y}$		W_1	W_2	(W_1-W_2)
d	$\boxed{\times}\ \boxed{-}$			W_1W_2	
90	$\boxed{\downarrow}\ \boxed{CNT}$		W_1W_2	(W_1-W_2)	
1	$\boxed{\div}\ \boxed{CNT}$			$(\div)$	
2	d		d^2		
3	↑			d^2	$(\div)$
4	$\boxed{2}$		2		
5	$\boxed{\times}$			$2d^2$	
6	$\boxed{0}$		0		
7	−			$(2d^2-0)$	
8	↓		$(2d^2-0)$	$(\div)$	
9	×			Σ_{34}	
a	0		0		
b	$\boxed{Acc+}\ \boxed{Acc-}$				
c	c		W_1^2		
d	↑			W_1^2	Σ''_{34}

Step	Key	Code	Display x	Display y	Display z
40	$\boxed{\sqrt{x}}\ \boxed{CNT}$		W_1	W_1^2	Σ''_{34}
1	$\boxed{\times}\ \boxed{CNT}$			W_1^3	
2	f		W_2^2		
3	↑			W_2^2	W_1^3
4	$\boxed{\sqrt{x}}\ \boxed{CNT}$		W_2		
5	$\boxed{\times}\ \boxed{CNT}$			W_2^3	
6	↓		W_2^3	W_1^3	
7	−			$(W_1^3-W_2^3)$	
8	Roll ↑	Table XIV	W_1^3	W_2^3	$(W_1^3-W_2^3)$
9	×			$W_1^3W_2^3$	
a	↓		$W_1^3W_2^3$	$(W_1^3-W_2^3)$	
b	÷			$(\div)$	
c	d		d^2		
d	↑			d^2	$(\div)$
60	$\boxed{0}$		0		
1	$\boxed{\times}$			0	
2	$\boxed{0}$		0		
3	$\boxed{-}$			0	
4	$\boxed{3}\ \boxed{2}$		3		
5	÷			$(\div)$	
6	↓		$(\div)$	$(\div)$	
7	×	Table XV		Σ''_{34}	
8	0		0		
9	Acc+				
a	c		W_1^2		
b	↑			W_1^2	Σ''_{34}
c	×			W_1^4	
d	$\boxed{\sqrt{x}}\ \boxed{CNT}$				

Storage

f	Note:
e	
d	Modular Program
c	Entries Entered
b	Are Those For
a	Z''_{132} and Z''_{133}.
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Title ______

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
c 0	X CNT		W_1	W_1^5	Σ
1	x^2		W_2^2		
2	↑			W_2^2	W_1^5
3	×			W_2^4	
4	X CNT		W_2		
5	X CNT			W_2^5	
6	↓		W_2^5	W_1^5	
7	−			$(W_1^5-W_2^5)$	
8	Roll ↑	X	W_1	W_2^5	$(W_1^5-W_2^5)$
9	×			$W_1W_2^5$	
a	↓		$W_1W_2^5$	$(W_1^5-W_2^5)$	
b	÷			$(\div)$	
c	d		d^2		
d	↑			d^2	$(\div)$
d 0	0		0		
1	×			0	
2	5 4		5		
3	÷			0	
4	↓		0	$(\div)$	
5	×			Z	
6	0		0		
7	ACC+				
8	e		Σ''		
9	↑			Z''	Z
a	Sub/Rtn				
b	END				
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

TABLE I

$(KW)^2_{max}$ PROGRAM ENTRIES

Step	Range		
	I	II	III
0.8	4	2	4
0.9	.	5	0
0.a	0	6	9
0.b	0	.	6

TABLE II

K^2 PROGRAM ENTRIES

Steps					Range		
					I	II	III
1.1	3.1	5.a	7.4	-3.2	8	6	1
1.2	3.2	5.b	7.5	-3.3	.	4	6

TABLE III

Σ_{III} PROGRAM ENTRIES (W_L < KW)

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
3.8	↑	$\sqrt{X}$	$\sqrt{X}$	↑	↑	↑	↑	↑
3.9	$\sqrt{X}$	↑	↑	$\sqrt{X}$	X	C	C	↑
3.a	X	C	↑	X	C	-	+	C
3.b	C	$\sqrt{X}$	C	C	↑	2	↓	-
3.c	↑	-	$\sqrt{X}$	↑	X	÷	ln(X)	Roll↑
3.d	$\sqrt{X}$	CONT	-	$\sqrt{X}$	↓	CONT	↑	X
4.0	X	CONT	Roll↑	X	-	CONT	2	2
4.1	↓	CONT	X	↓	4	CONT	÷	X
4.2	-	CONT	↓	-	÷	CONT	CONT	↓
4.3	3	CONT	+	Roll↑	CONT	CONT	CONT	+
4.4	+	CONT	CONT	X	CONT	CONT	CONT	CONT
4.5	CONT	CONT	CONT	3	CONT	CONT	CONT	CONT
4.6	CONT	CONT	CONT	X	CONT	CONT	CONT	CONT
4.7	CONT	CONT	CONT	↓	CONT	CONT	CONT	CONT
4.8	CONT	CONT	CONT	+	CONT	CONT	CONT	CONT

TABLE IV

Σ_I PROGRAM ENTRIES (W_L > KW)

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
6.1	7	7 3	.	2	6	6 1	1	1
6.2	.	↑	7	1	3	3 5	6	5
6.3	2	C	5	↑	↑	↑	ln(X)	↑
6.4	0	$\sqrt{X}$	↑	6	C	C	↑	3
6.5	9	X	C	4	X	X	2	2
6.6	1	CONT	$\sqrt{X}$	÷	X	2	÷	÷
6.7	4	CONT	÷	C	4	÷	CONT	C
6.8	0	CONT	CONT	↑	÷	CONT	CONT	÷
6.9	0	CONT	CONT	$\sqrt{X}$	CONT	CONT	CONT	CONT
6.a	0	CONT	CONT	X	CONT	CONT	CONT	CONT
6.b	0	CONT	CONT	↓	CONT	CONT	CONT	CONT
6.c	0	CONT	CONT	÷	CONT	CONT	CONT	CONT
6.d	0	CONT	CONT	CONT	CONT	CONT	CONT	CONT

TABLE V

W^2_{min} PROGRAM ENTRIES

Step	Range					
	I		II		III	
	fn/in		fn/in		fn/in	
	1	2	10	20	20	50
7.7	.	.	.	.	.	.
7.8	3	0	0	0	3	0
7.9	0	7	2	0	9	6
7.a	5	6	4	6	0	2
7.b	1	2	4	1	6	5
7.c	7	9	1	0	2	0
7.d	5	3	4	3	5	0
8.a	8	9	0	5	0	0
8.b	0	5	8	2	0	0

TABLE VI

R^2 PROGRAM ENTRIES

Step	R^2			
	4	2	$\sqrt{2}$	$+\sqrt{2}$
9.3	4	2	2	2
9.4	.	.	$\sqrt{X}$	$\sqrt{X}$
9.5	0	0	CONT	$\sqrt{X}$
9.6	0	0	CONT	CONT

TABLE VII

W CORRECTION PROGRAM ENTRIES

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
a.4	↑	$\sqrt{X}$	$\sqrt{X}$	↑	↑	÷	CONT	X
a.5	$\sqrt{X}$	÷	X	$\sqrt{X}$	X	CONT	CONT	CONT
a.6	X	CONT	CONT	X	↓	CONT	CONT	CONT
a.7	↓	CONT	CONT	↓	÷	CONT	CONT	CONT
a.8	÷	CONT	CONT	X	CONT	CONT	CONT	CONT

TABLE VIII

Yscale PROGRAM ENTRIES (LOG SWEEP)
(STEPS a.9 THROUGH b.1)

in-lb/in	Range I	Range II	Range III
10^9	.000204	.004611	.036890
2×10^8	.001018	.023060	.184500
10^8	.002036	.046110	.368900
5×10^7	.004072	.092220	.737800
2×10^7	.010180	.230600	1.84500
10^7	.020360	.461100	3.68900
5×10^6	.040720	.922200	7.37800
2.5×10^6	.081440	1.84400	14.7600
2×10^6	.101800	2.30600	18.4500
10^6	.203600	4.61100	36.8900
5×10^5	.407200	9.22200	73.7800
2.5×10^5	.814400	18.4400	147.600
2×10^5	1.01800	23.0600	184.500
10^5	2.03600	46.1100	368.900
5×10^4	4.07200	92.2200	737.800
2.5×10^4	8.14400	184.400	1476.00
2×10^4	10.1800	230.600	1845.00
10^4	20.3600	461.100	3689.00
5×10^3	40.7200	922.200	7378.00
2.5×10^3	81.4400	1844.00	14760.0
2×10^3	101.800	2306.00	18450.0
10^3	203.600	4611.00	36890.0
500	407.200	9222.00	73780.0
250	814.400	18440.0	147600.
200	1018.00	23060.0	184500.

TABLE IX

Yscale PROGRAM ENTRIES (LINEAR SWEEP) (STEPS a.9 THROUGH b.1)

in-lb/in	Range I	Range II	Range III
10^9	.000010	.000670	.042880
2×10^8	.000052	.003350	.214400
10^8	.000105	.006700	.428800
5×10^7	.000209	.013400	.857600
2×10^7	.000524	.033500	2.14400
10^7	.001047	.067000	4.28800
5×10^6	.002094	.134000	8.57600
2.5×10^6	.004188	.268000	17.1500
2×10^6	.005235	.335000	21.4400
10^6	.010470	.670000	42.8800
5×10^5	.020940	1.34000	85.7600
2.5×10^5	.041880	2.68000	171.500
2×10^5	.052350	3.35000	214.400
10^5	.104700	6.70000	428.800
5×10^4	.209400	13.4000	857.600
2.5×10^4	.418800	26.8000	1715.00
2×10^4	.523500	33.5000	2144.00
10^4	1.04700	67.0000	4288.00
5×10^3	2.09400	134.000	8576.00
2.5×10^3	4.18800	268.000	17150.0
2×10^3	5.23500	335.000	21440.0
10^3	10.4700	670.000	42880.0
500	20.9400	1340.00	85760.0
250	41.8800	2680.00	171500.
200	52.3500	3350.00	214400.
100	104.700		
.50	209.400		

TABLE X

X_{scale} PROGRAM ENTRIES
(STEPS b.3 THROUGH b.9, AND-2.3 THROUGH-2.9)

fn/in	Range		
	I	II	III
50			1250.00
20		390.625	3125.00
10		781.250	
2	1381.07		
1	2762.14		

TABLE XI

Σ''_{MN} PROGRAM ENTRIES

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
-4.1	+				X			
-5.1	X				+			
-5.8	3	5	7	9	1	2	3	4
-5.9	X	X	X	X	X	X	X	X
-5.a	1	1	3	5	1	0	1	2
-5.b	+	-	-	-	+	+	-	-
-5.c	4	4	4	4	2	2	2	2

TABLE XII

Σ''_{OP} PROGRAM ENTRIES

Step	Sweep	
	Log	Linear
-6.4	f	1
-7.3	C	1

TABLE XIII

Σ''_{BA} PROGRAM ENTRIES

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
-8.7	↑				ln(X)			
-8.b	-				ln(X)			
-8.c	Roll↑				XY			
-8.d	X				-			
-9.0	↓				CONT			
-9.1	÷				CONT			
-9.4	0	2	3	4	0	2	3	4
-9.5	X	X	X	X	X	X	X	X
-9.6	0	0	1	2	0	0	1	2
-9.b	ACC +				ACC -			

T A B L E X I V

Σ''_{DC} PROGRAM ENTRIES

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
-a.0		$\sqrt{X}$				CONT		
-a.1		X				CONT		
-a.4		$\sqrt{X}$				CONT		
-a.5		X				CONT		
-b.0	0	0	2	3	0	0	2	3
-b.1	X	X	X	X	X	X	X	X
-b.2	0	0	0	1	0	0	0	1
-b.3	-	-	-	-	-	-	-	-
-b.4	3	3	3	3	2	2	2	2

T A B L E X V

Σ''_{LJ} PROGRAM ENTRIES

Step	Log Sweep				Linear Sweep			
	S1	S2/S5	S3	S6	S7	S8/S11	S9	S12
-b.d		$\sqrt{X}$				CONT		
-€.0		X				CONT		
-c.4		$\sqrt{X}$				CONT		
-c.5		X				CONT		
-d.0	0	0	0	2	0	0	0	2
-d.1	X	X	X	X	X	X	X	X
-d.2	5	5	5	5	4	4	4	4

TABLE XVI

CURVE PLOTTING PARAMETERS FOR RANGES I AND II

Range	E''_X	Fig	Family	fn Min	fn Max	Scaling Rate Hor (fn/in)	Scaling Rate Vert (in-lb/in)
I	E''_{S1}	4	1	0.9756	20	2	$2(10^4)$
		5	2		10	1	500
		6	2		20	2	$2.5(10^3)$
	E''_{S7}	7	1	0.9766	20	2	$2.5(10^3)$
		8	2		10	1	50
		9	2		20	2	250
II	E''_{S2}	10	1	7.8125	200	20	$2(10^5)$
		11	2		100	10	$1(10^4)$
		12	2		200	20	$2.5(10^4)$
	E''_{S8}	13	1	7.8125	200	20	$2(10^5)$
		14	2		100	10	$1(10^4)$
		15	2		200	20	$2.5(10^4)$

TABLE XVII

CURVE PLOTTING PARAMETERS FOR RANGE III

Range	E''_X	Fig	Family	fn		Scaling Rate	
				Min	Max	Hor (fn/in)	Vert (in-lb/in)
III	E''_{S5}	16	1	7.8125	500	50	$5(10^5)$
		17	2		200	20	$5(10^4)$
		18	2		500	50	$1(10^5)$
	E''_{S11}	19	1	7.8125	500	50	$2.5(10^6)$
		20	2		200	20	$1(10^5)$
		21	2		500	50	$5(10^5)$
	E''_{S3}	22	1	7.8125	500	50	$2(10^5)$
		23	2		200	20	$1(10^4)$
		24	2		500	50	$2.5(10^4)$
	E''_{S9}	25	1	7.8125	500	50	$2(10^5)$
		26	2		200	20	$2.5(10^4)$
		27	2		500	50	$5(10^4)$
	E''_{S6}	28	1	7.8125	500	50	$1(10^5)$
		29	2		200	20	$5(10^3)$
		30	2		500	50	$2(10^4)$
	E''_{S12}	31	1	7.8125	500	50	$2(10^5)$
		32	2		200	20	$1(10^4)$
		33	2		500	50	$2.5(10^4)$

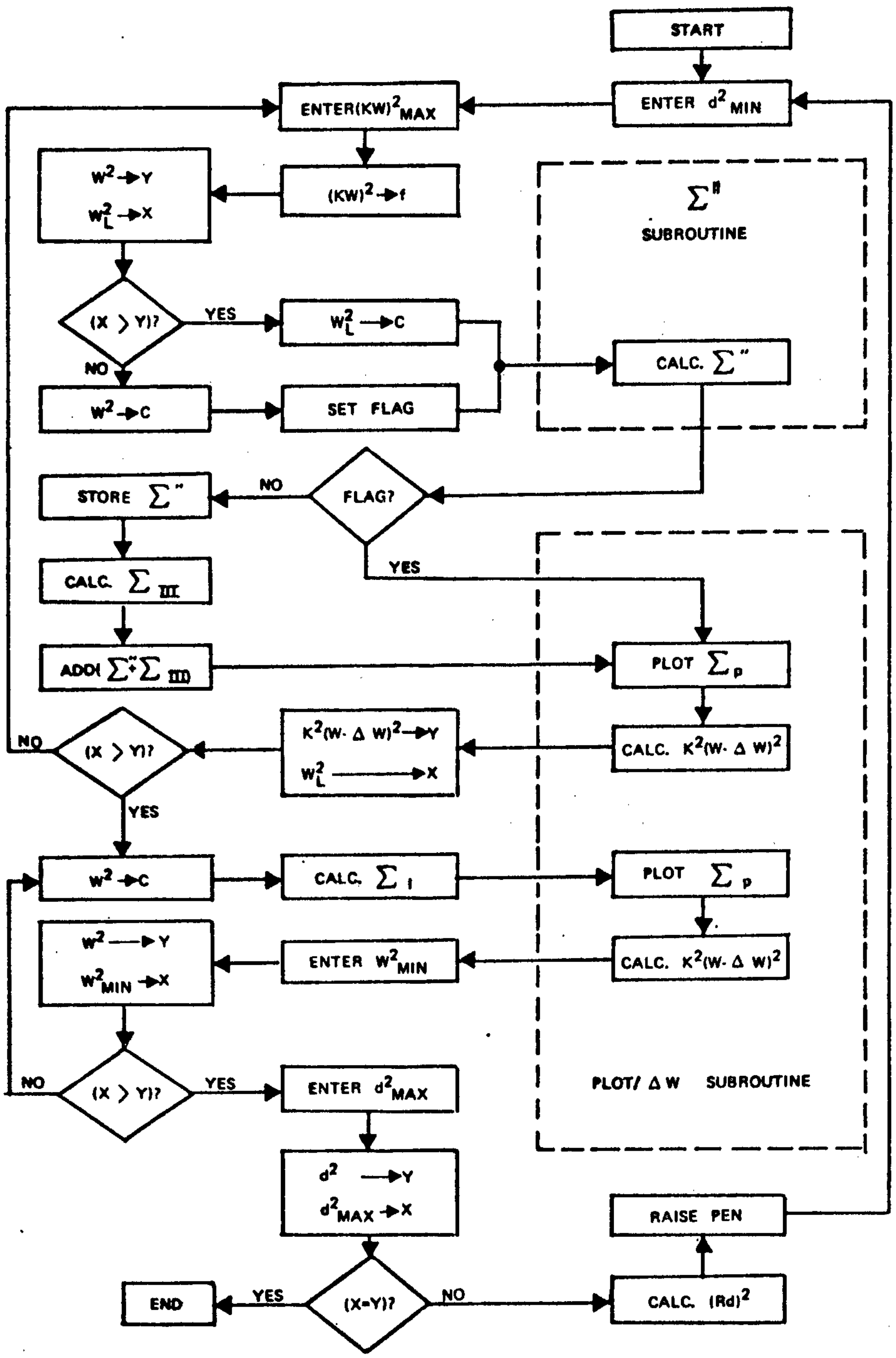

FIGURE 1 - E_x'' Plotting Program Block Diagram

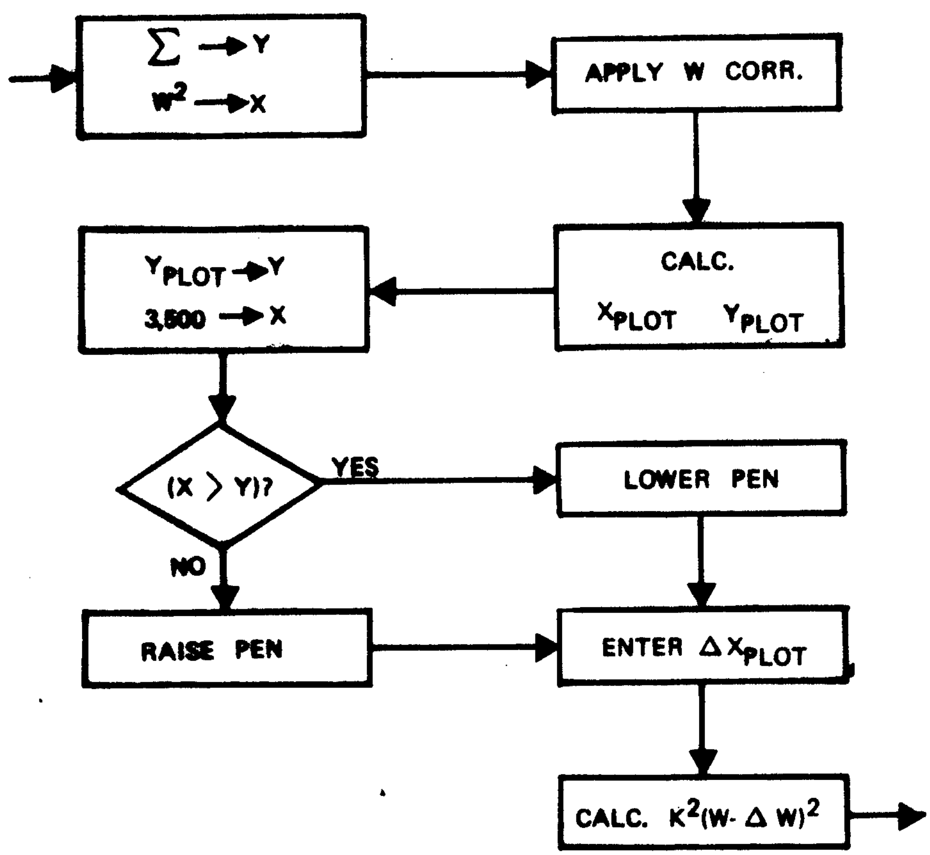

FIGURE 2 - PLOT/ΔW Subroutine Block Diagram

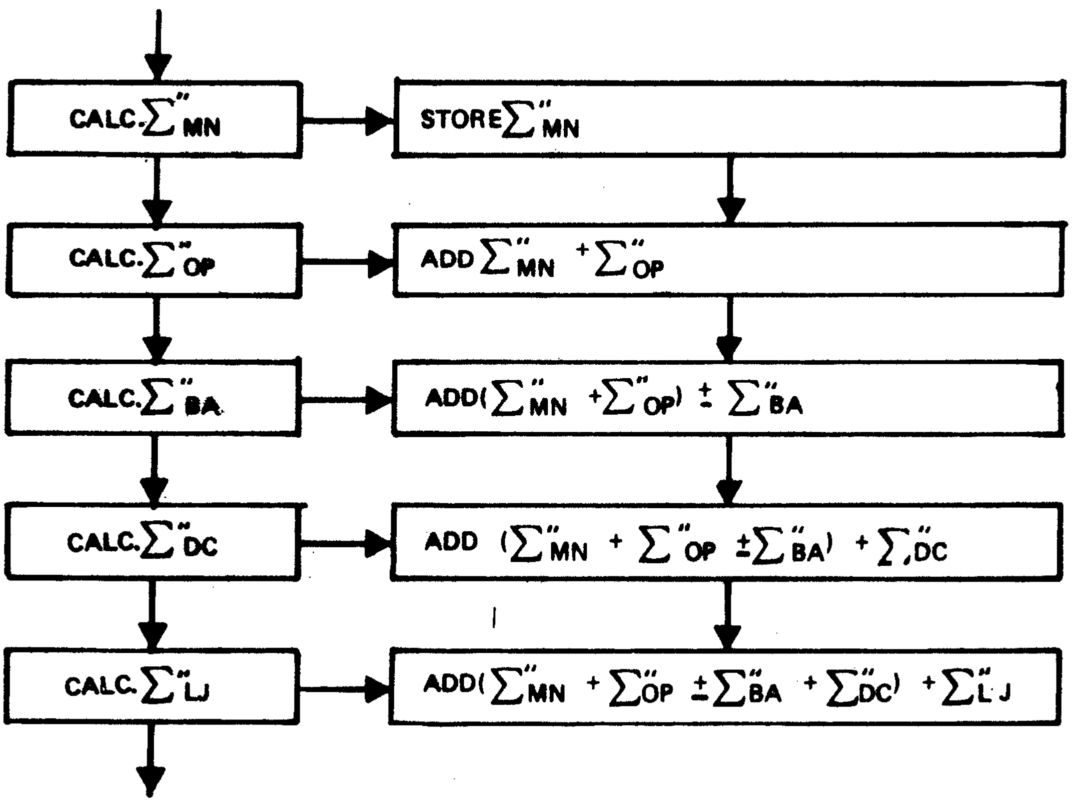

FIGURE 3 - Σ" Subroutine Block Diagram

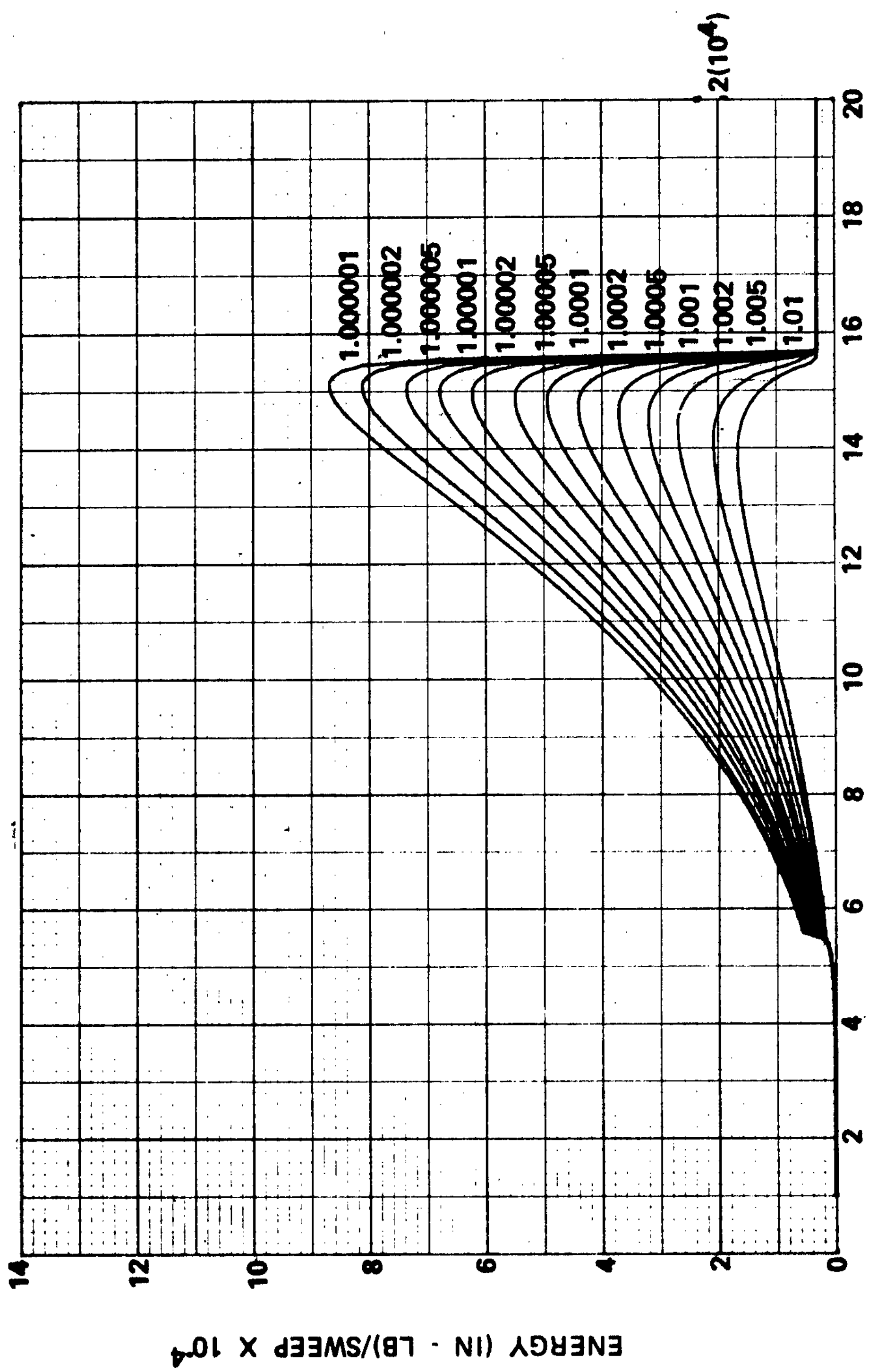

FIGURE 4 - E''_{S1} Versus fn for Family 1 d Values

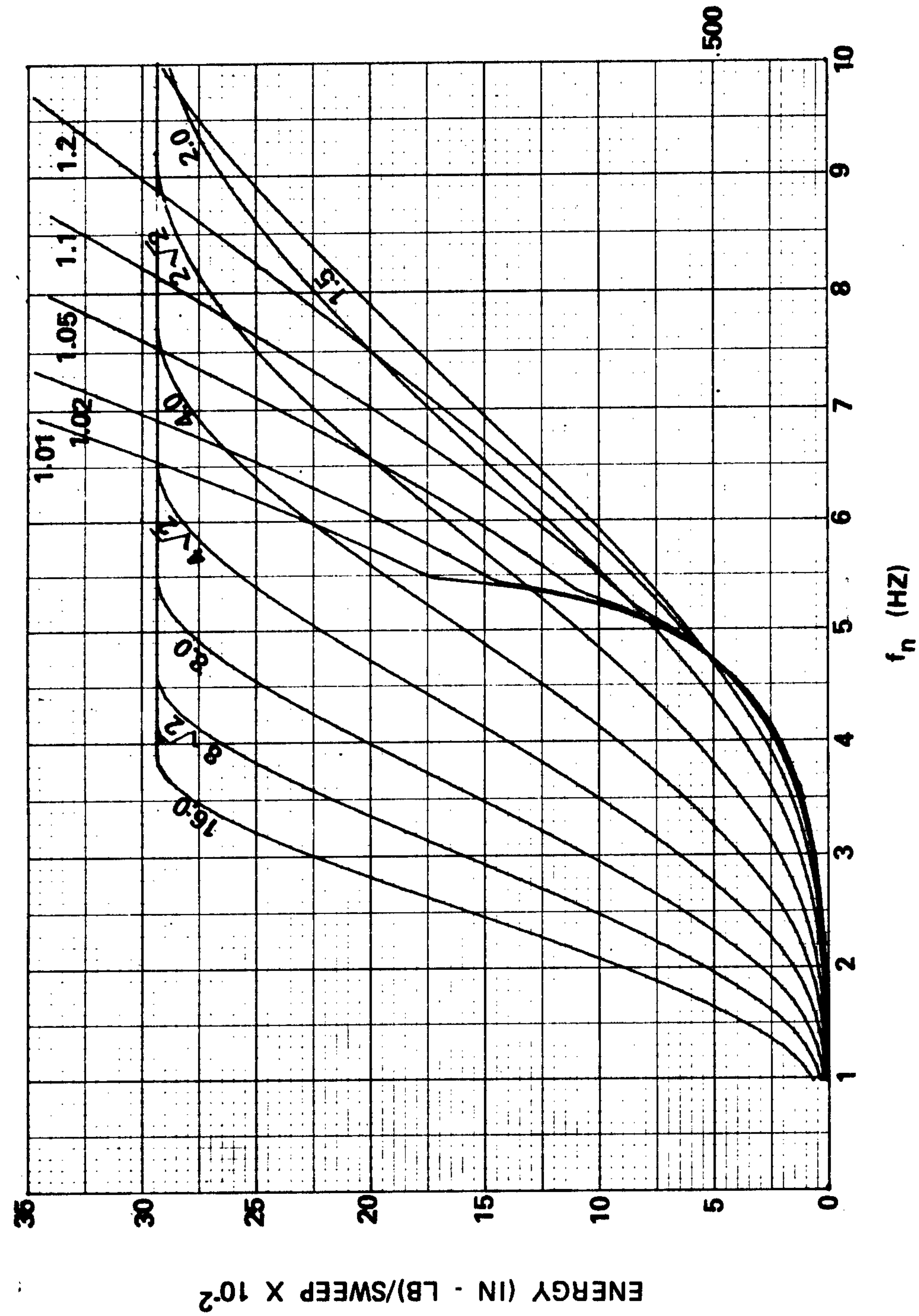

FIGURE 5 - E''_{S1} Versus fn for Family 2 d Values (Expanded)

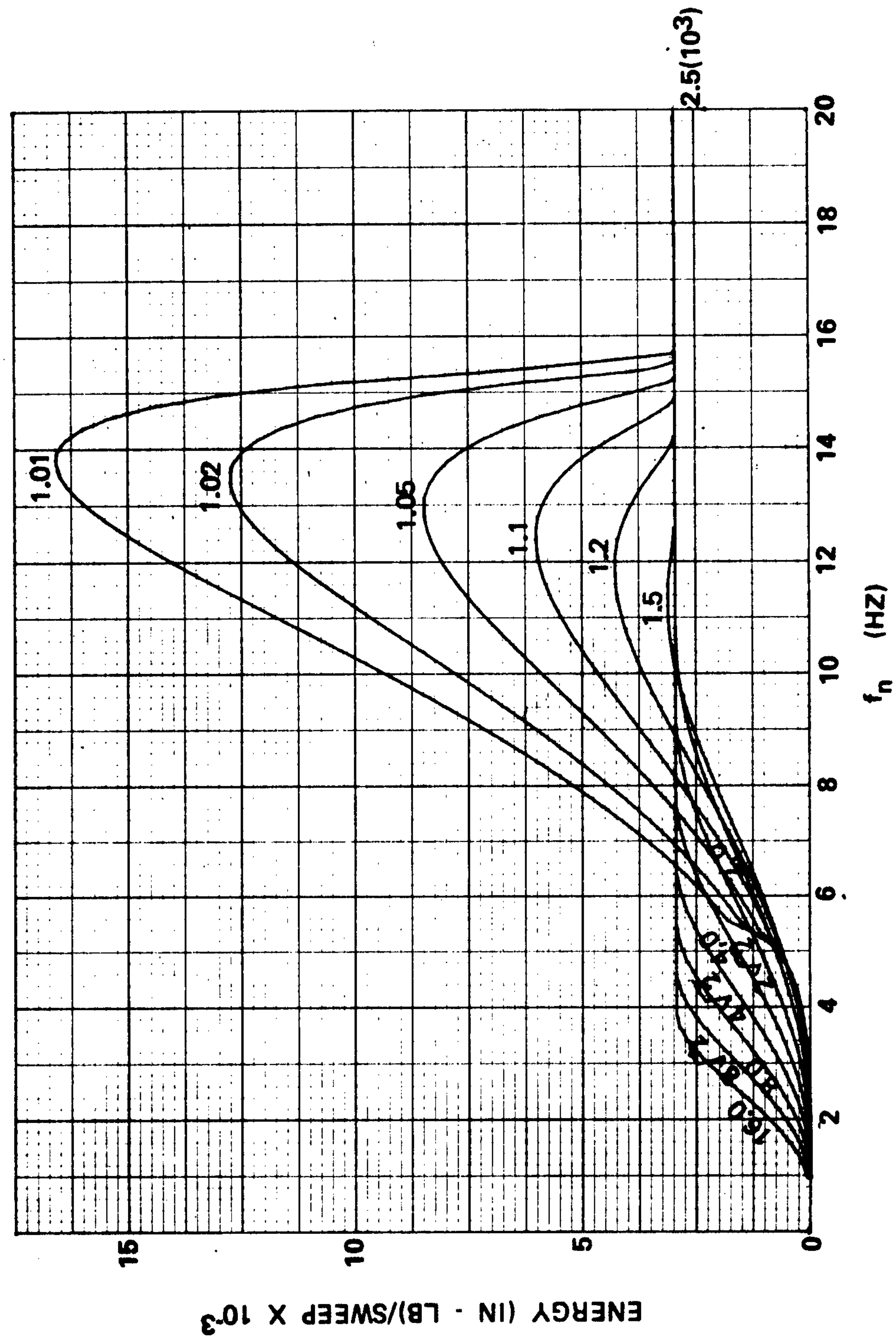

FIGURE 6 - E''_{S1} Versus fn for Family 2 d Values

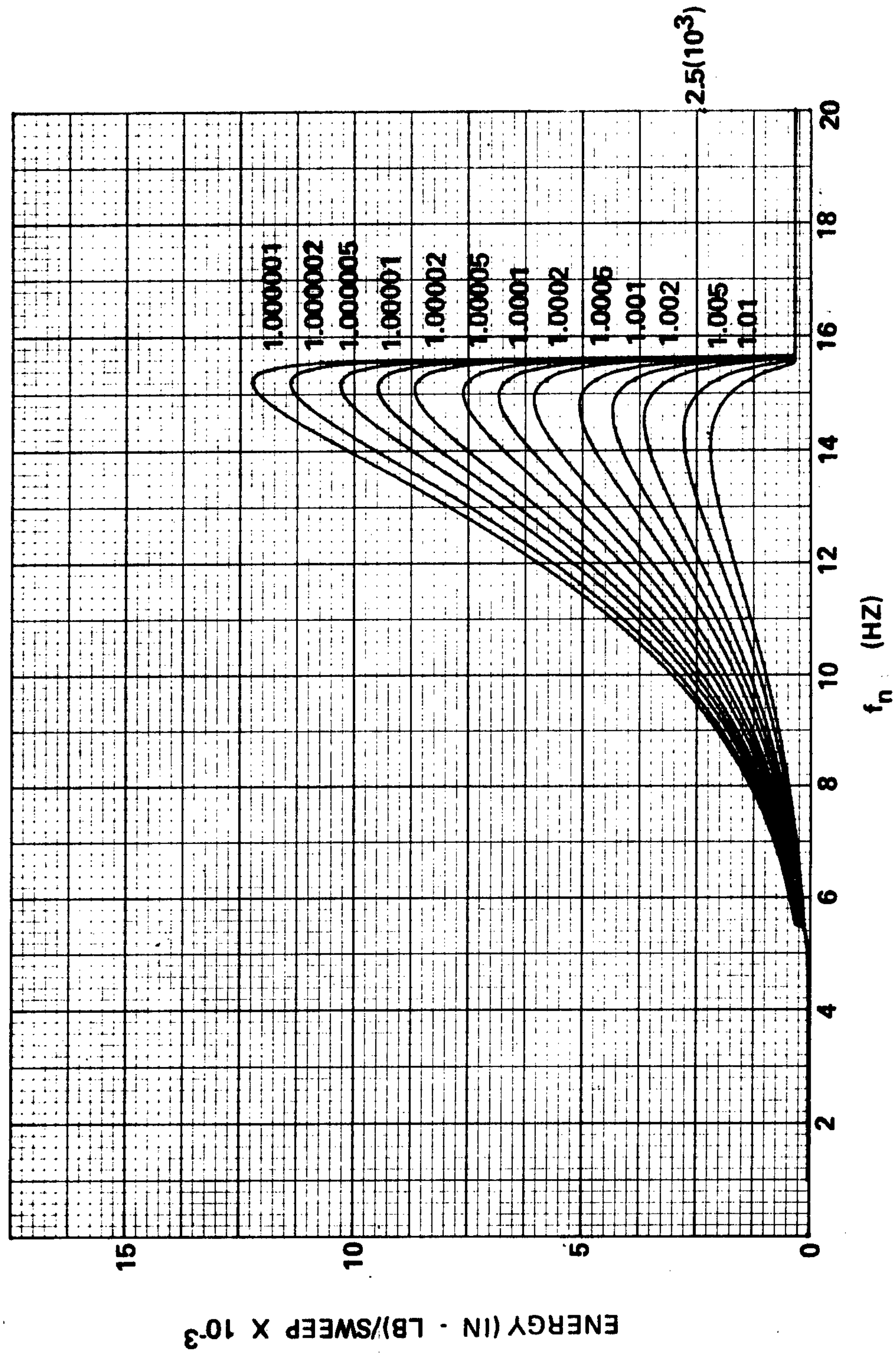

FIGURE 7 - E''_{S7} Versus fn for Family 1 d Values

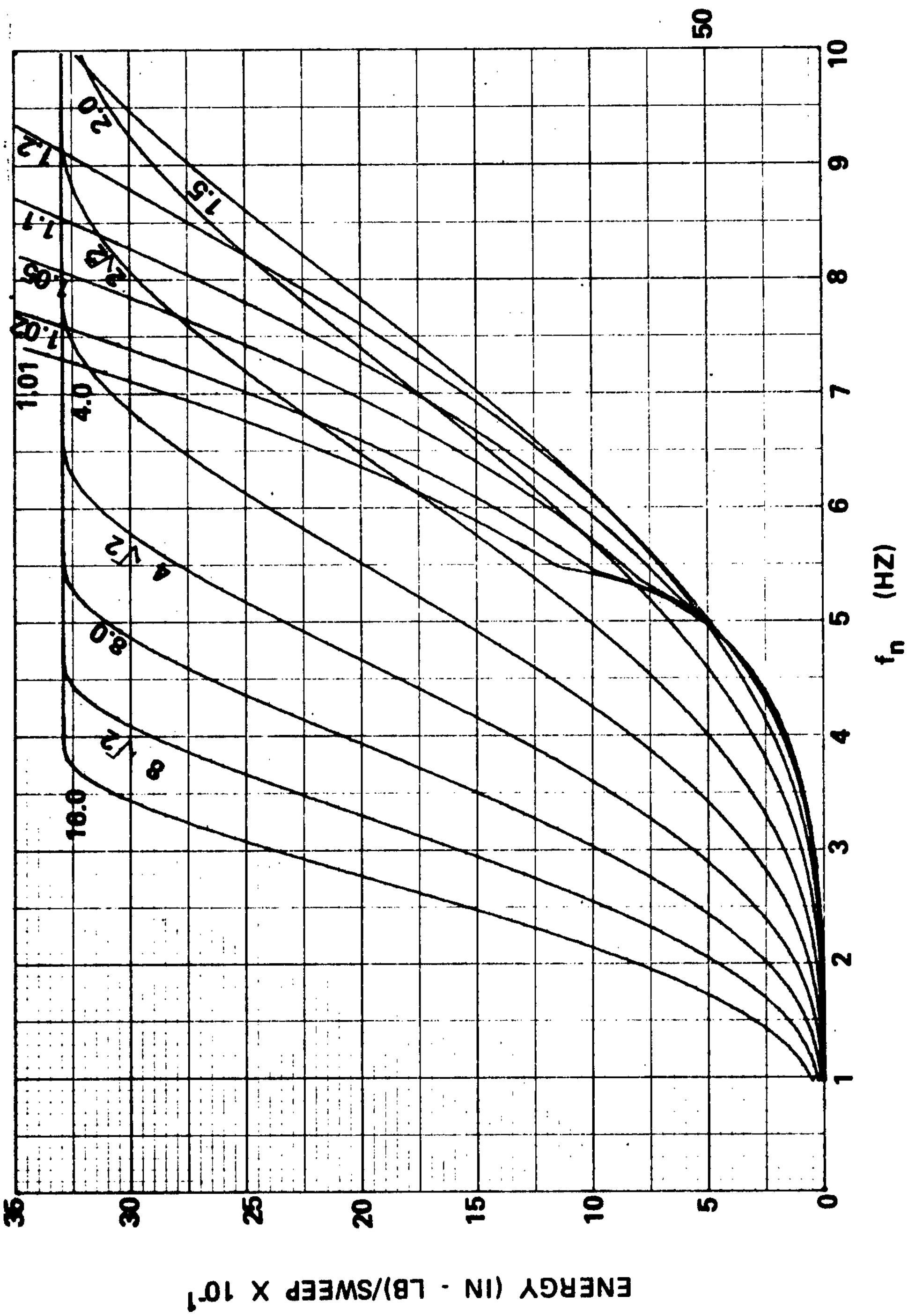

FIGURE 8 - E''_{S7} Versus fn for Family 2 d Values (Expanded)

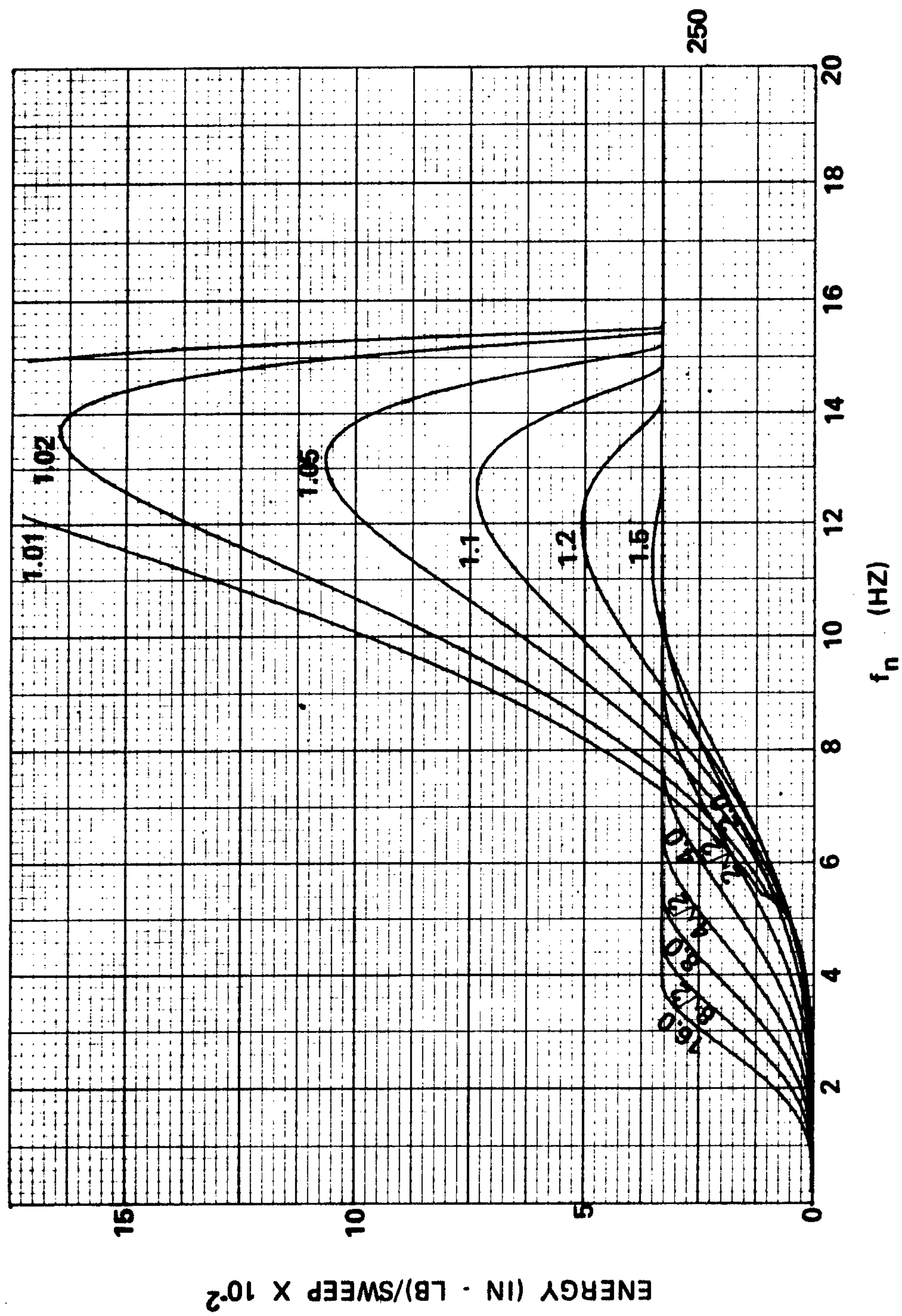

FIGURE 9 - E''_{S7} Versus fn for Family 2 d Values

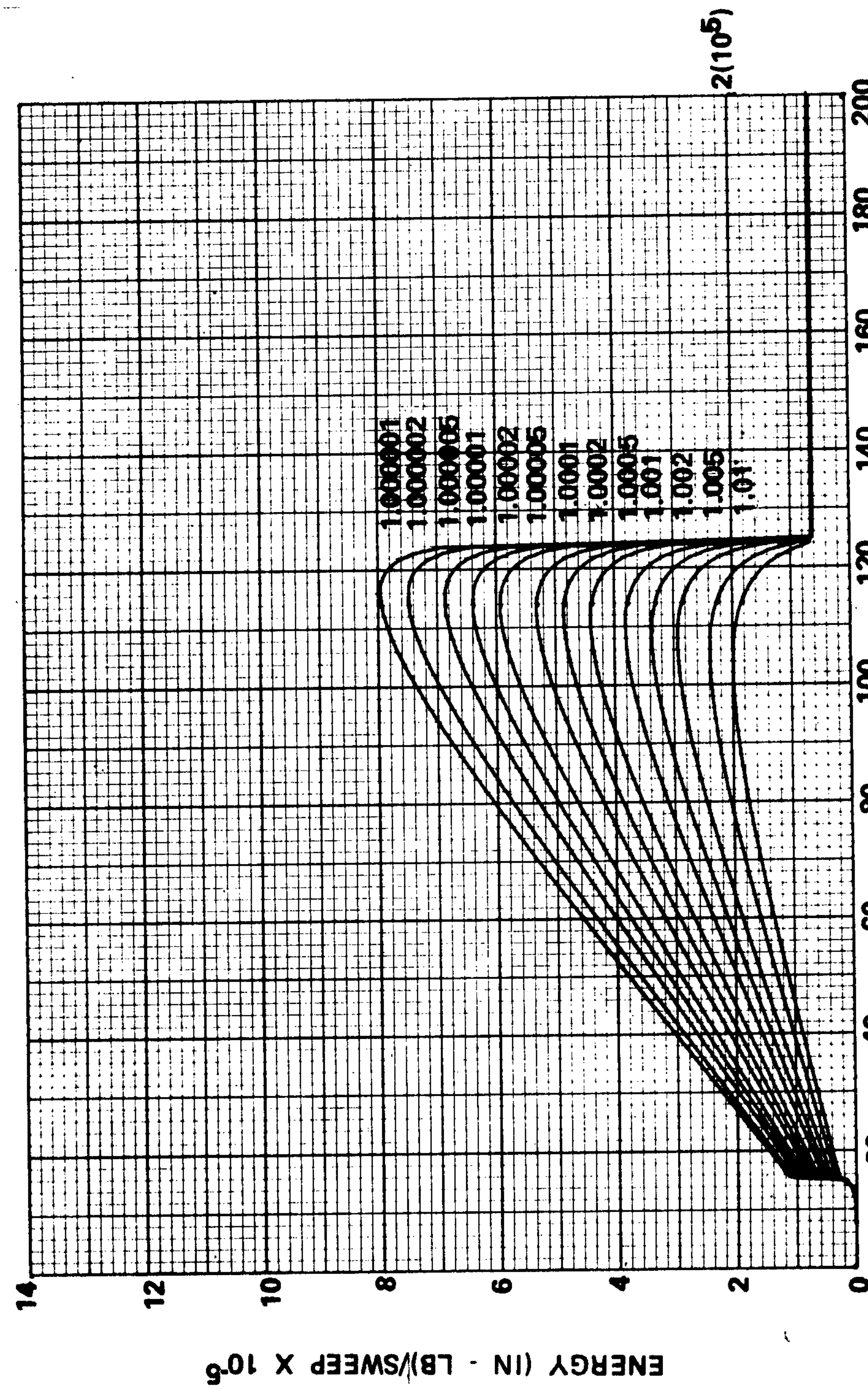

FIGURE 10 - E''_{S2} Versus fn for Family 1 d Values

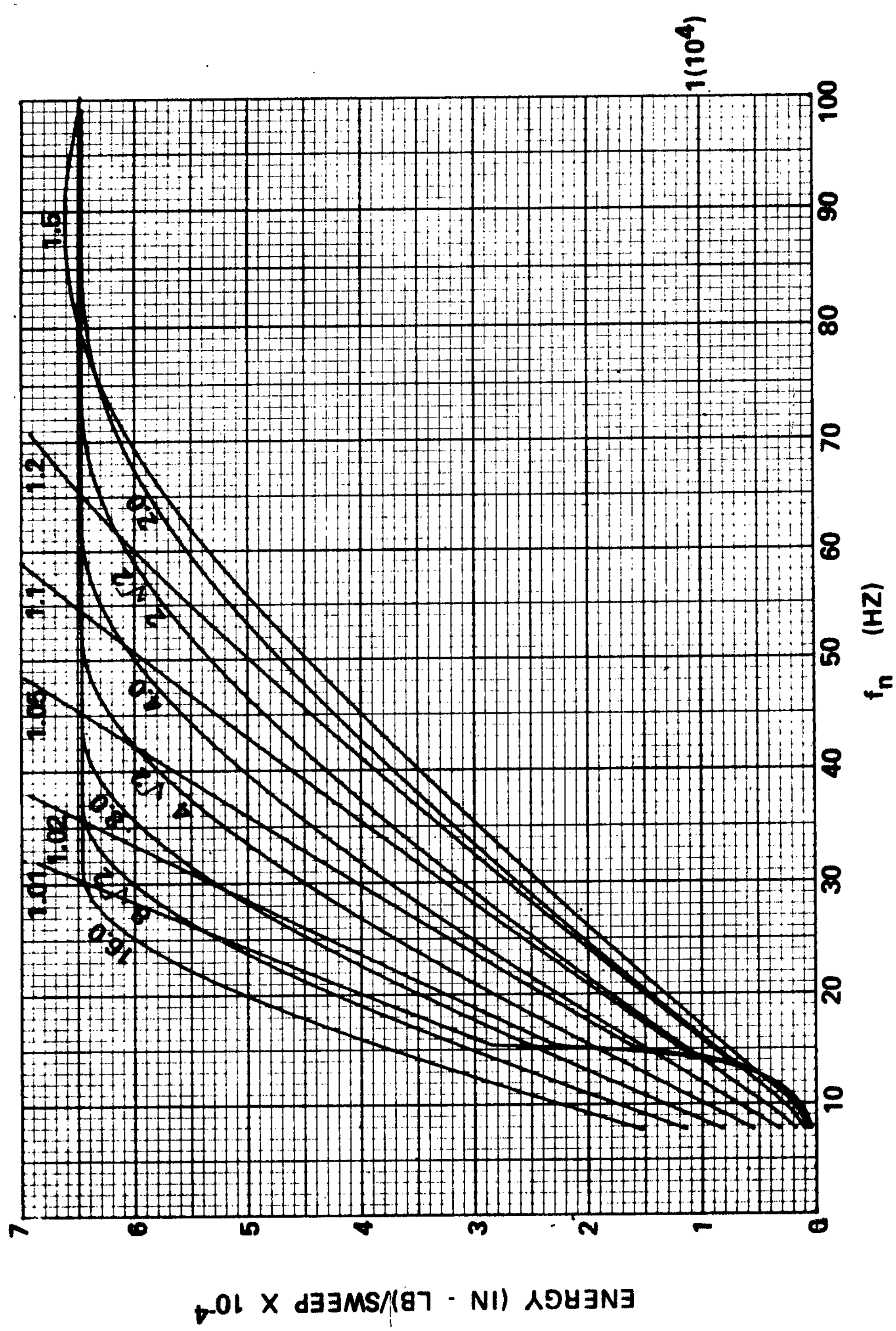

FIGURE 11 - E''_{S2} Versus fn for Family 2 d Values (Expanded)

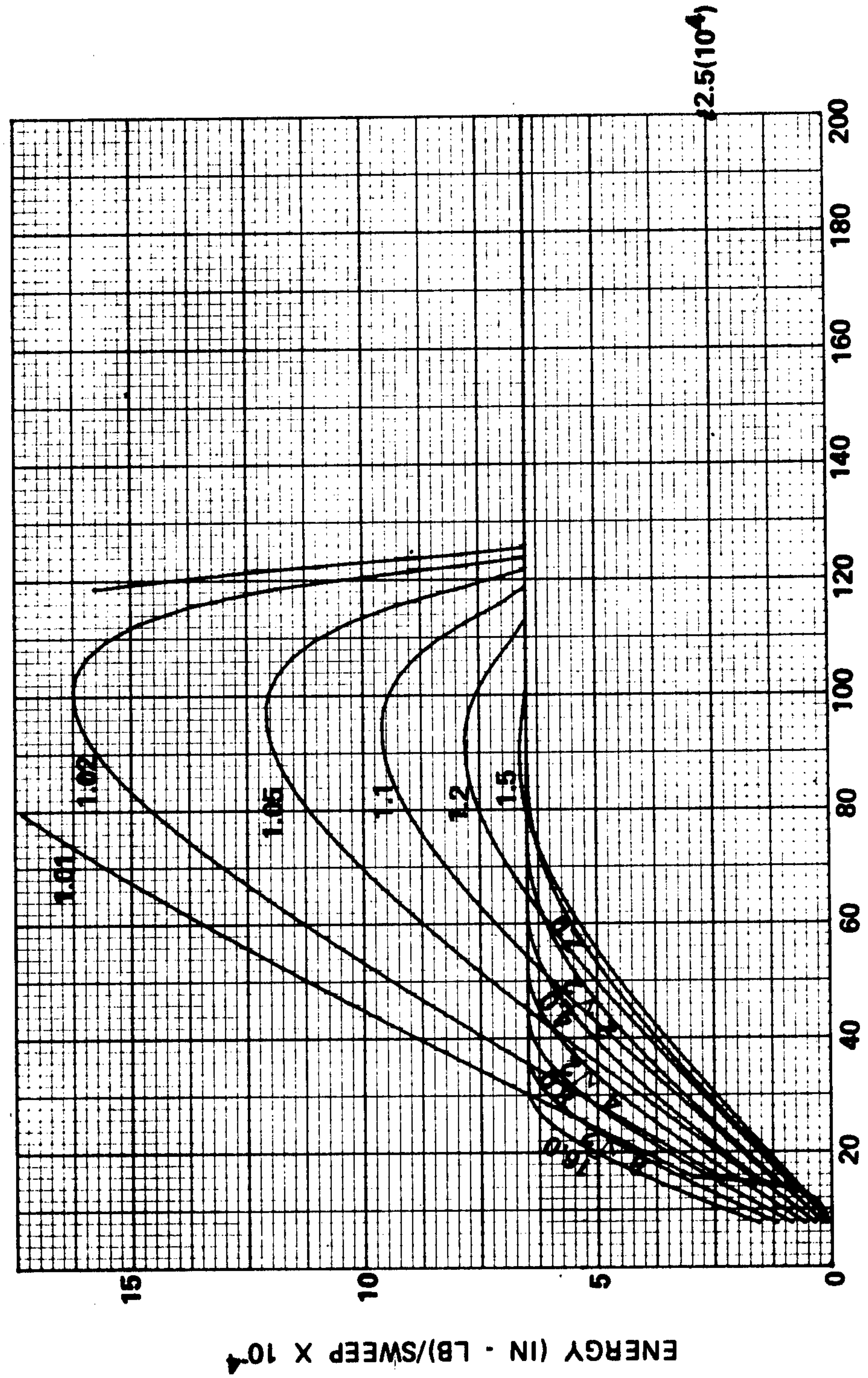

FIGURE 12 - E''_{S2} Versus fn for Family 2 d Values

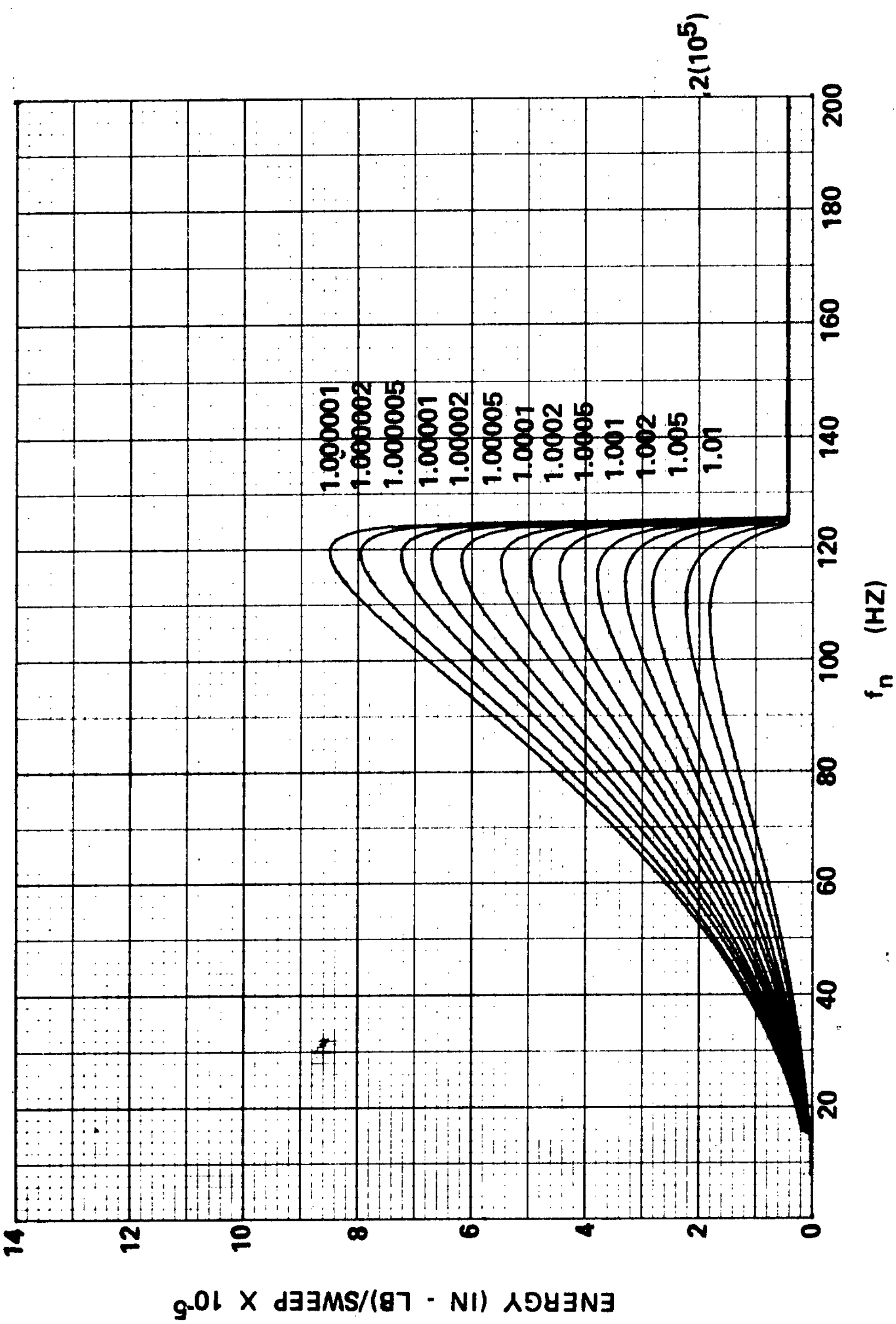

FIGURE 13 - E''_{S8} Versus fn for Family 1 d Values

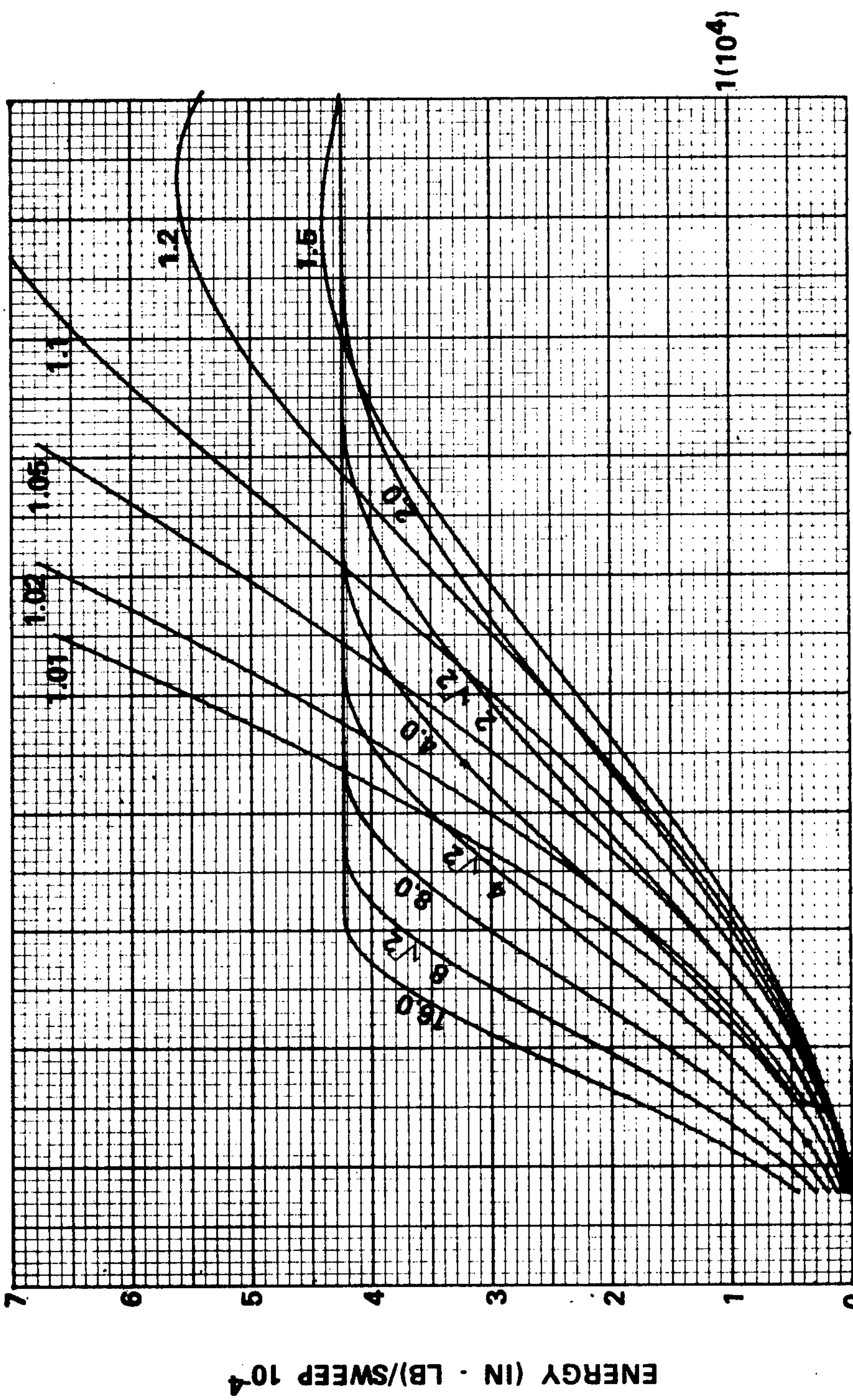

FIGURE 14 - E''_{S8} Versus fn for Family 2 d Values (Expanded)

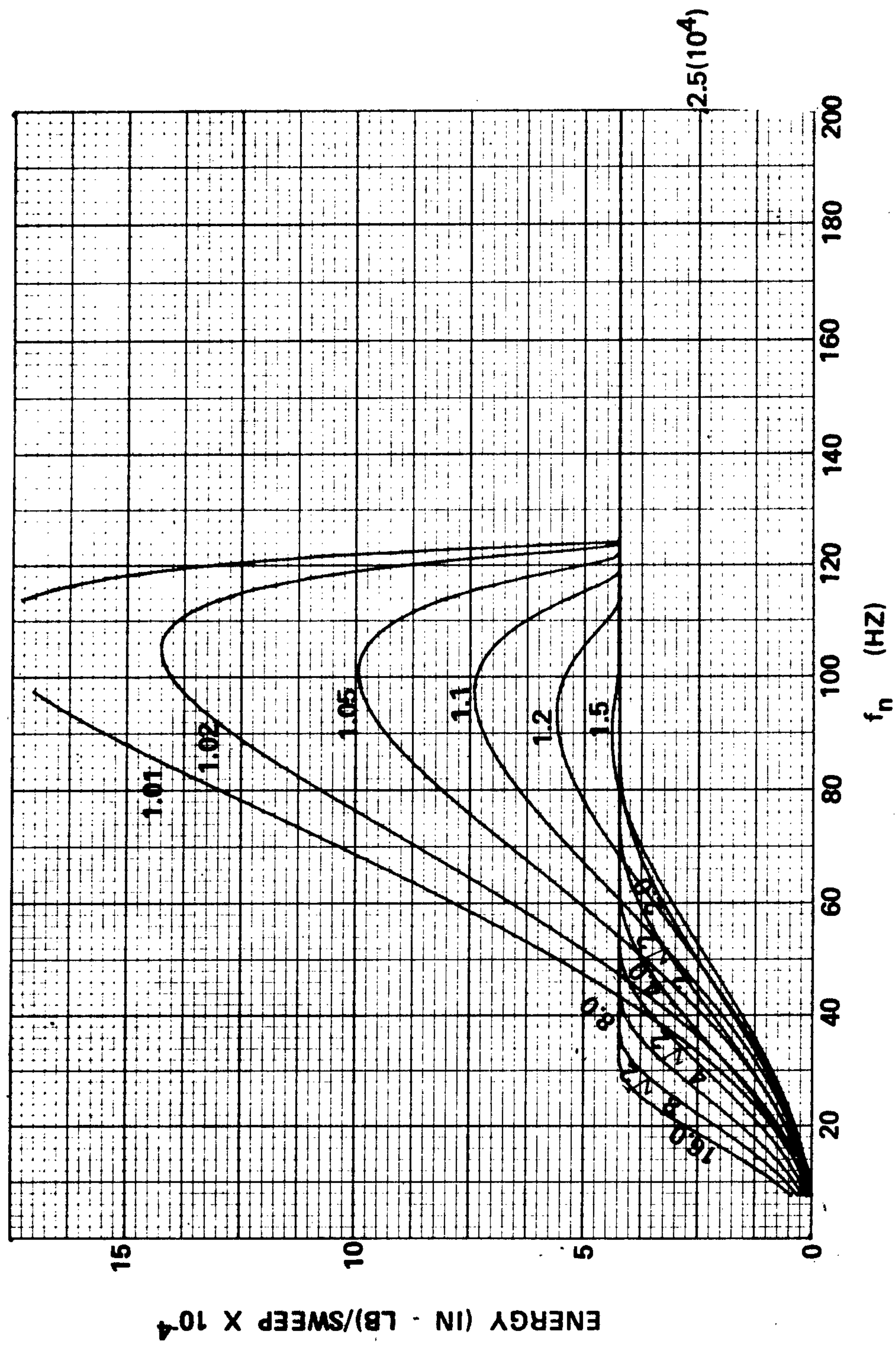

FIGURE 15 - E''_{S8} Versus fn for Family 2 d Values

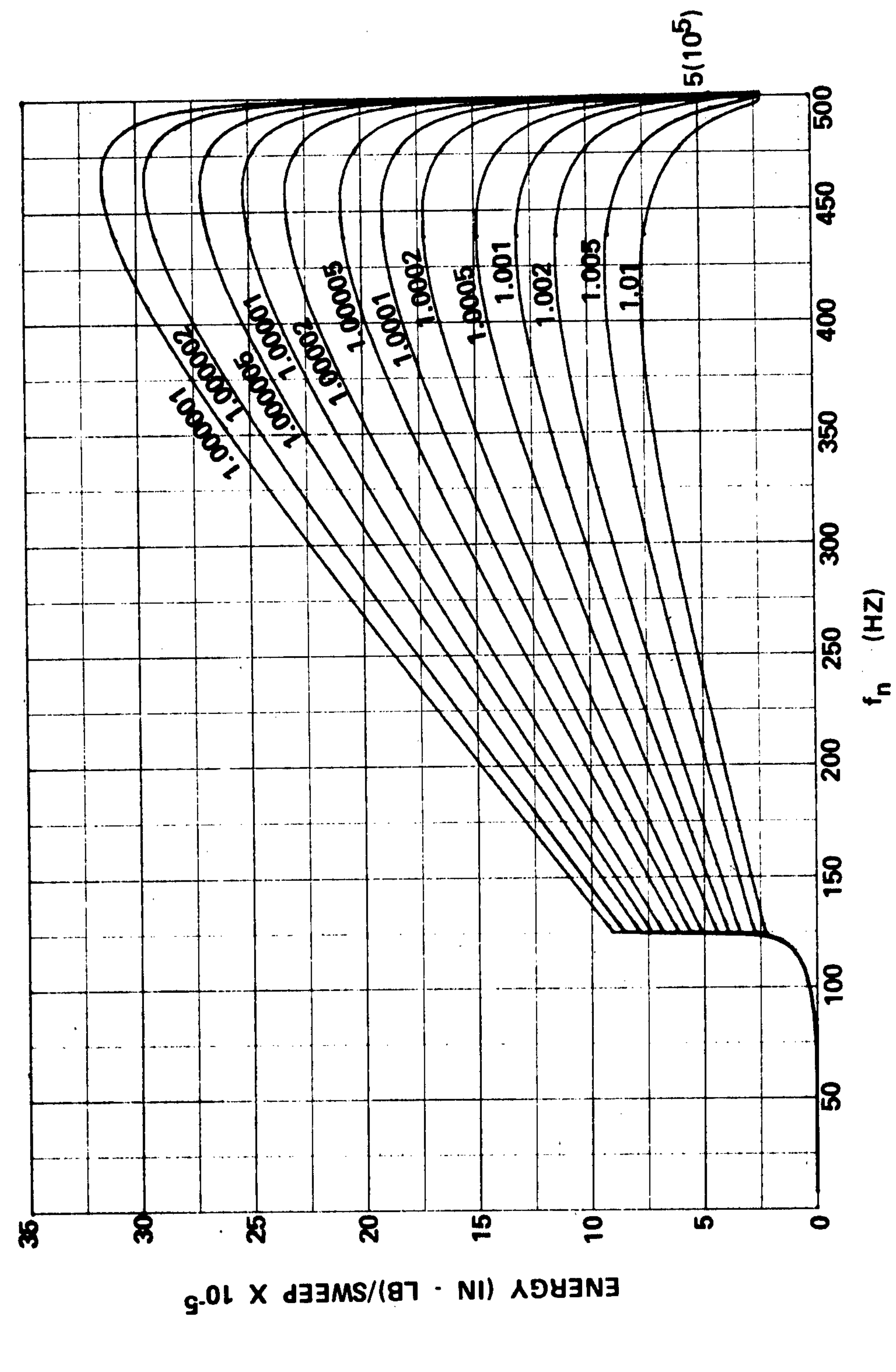

FIGURE 16 - E''_{S5} Versus fn for Family 1 d Values

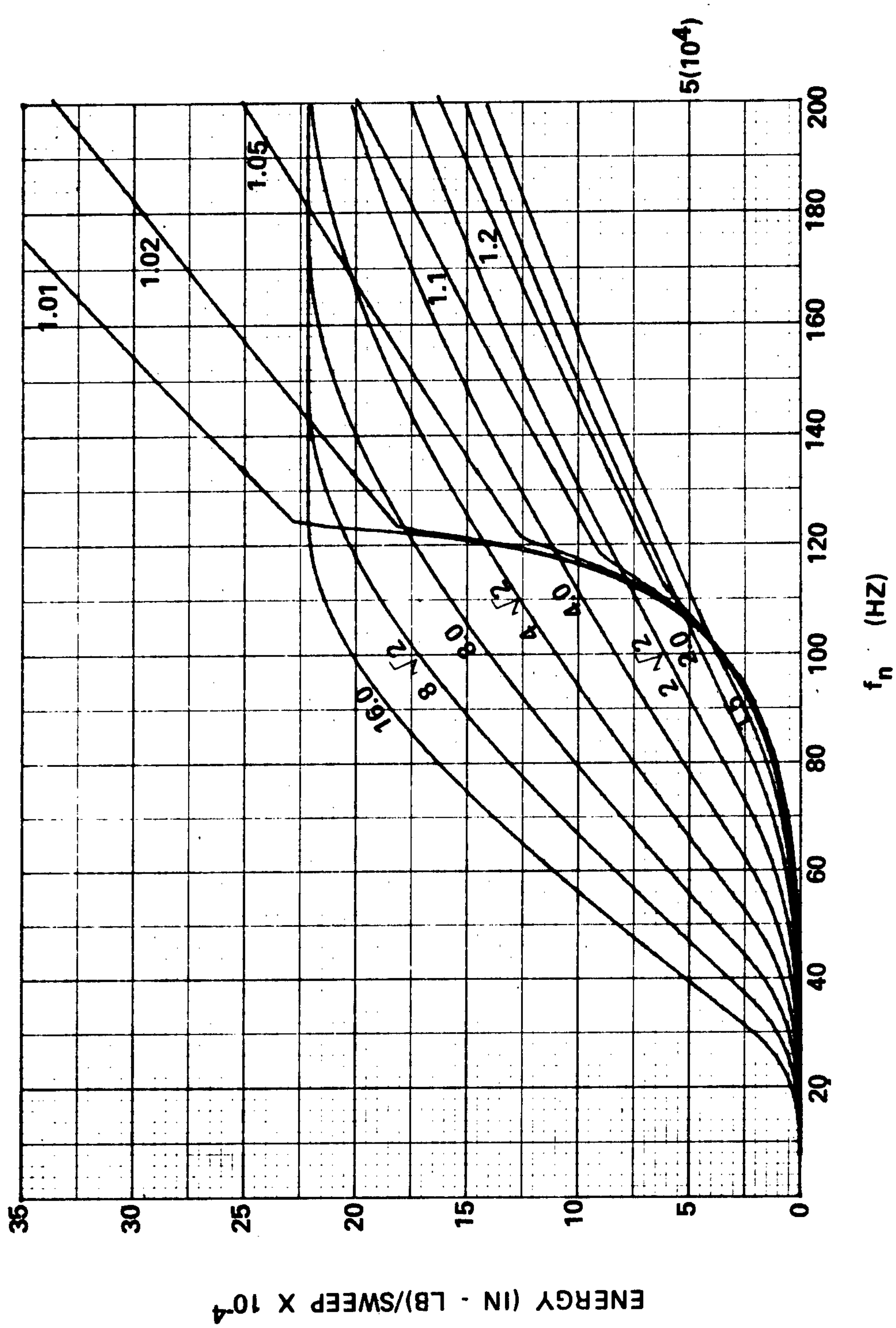

FIGURE 17 - E''_{S5} Versus fn for Family 2 d Values (Expanded)

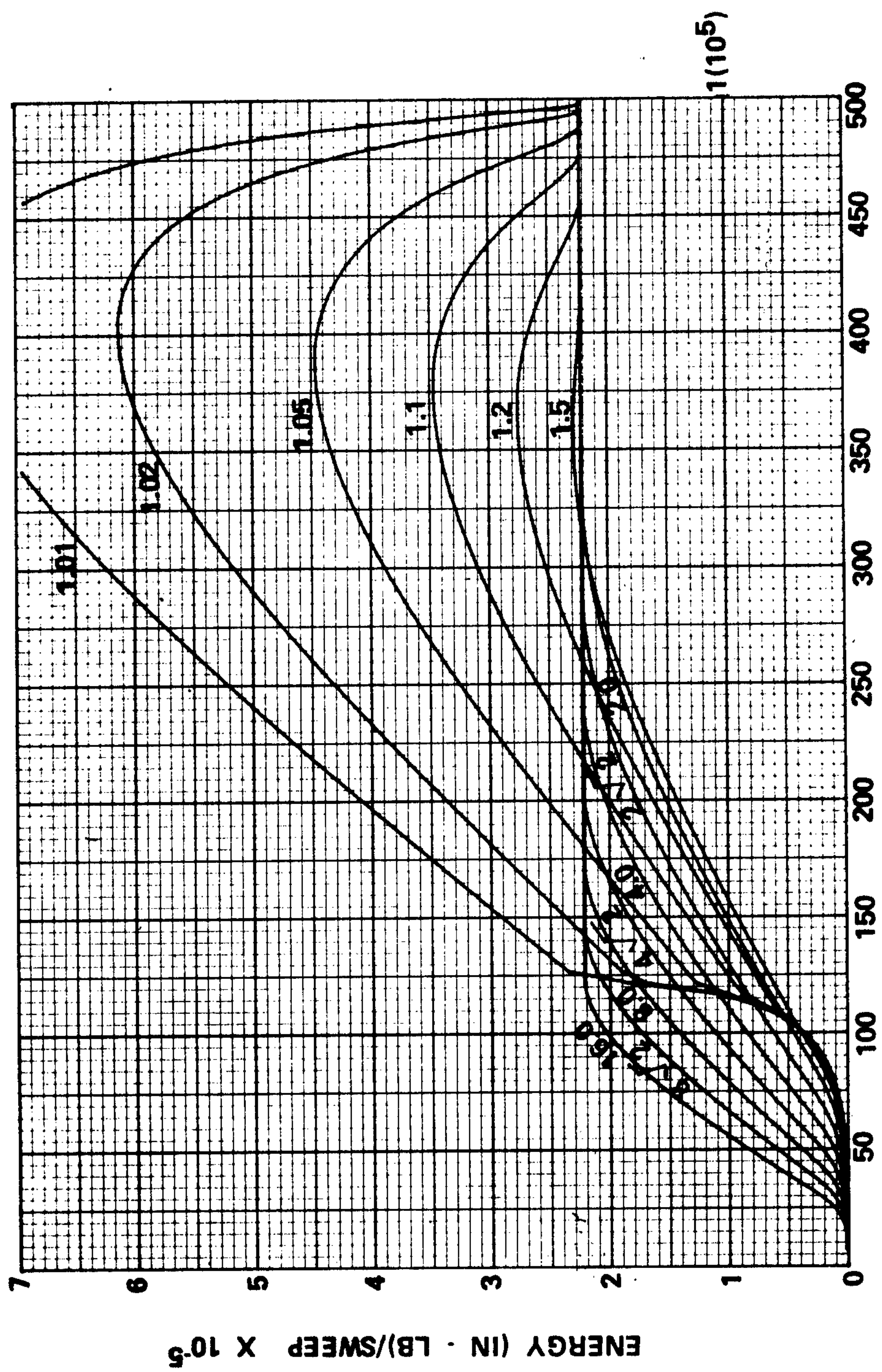

FIGURE 18 - E''_{S5} Versus fn for Family 2 d Values

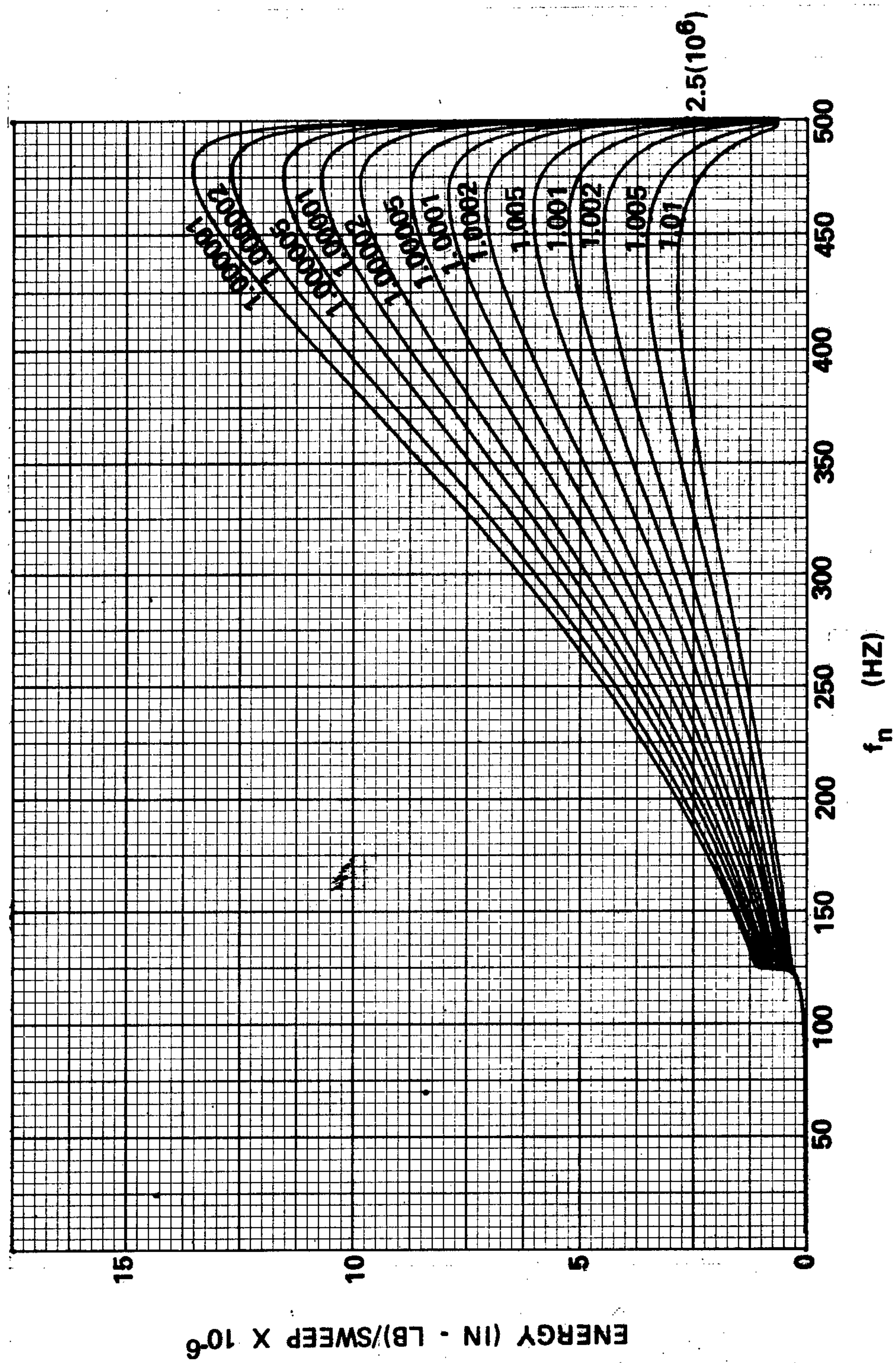

FIGURE 19 - E''_{S11} Versus fn for Family 1 d Values

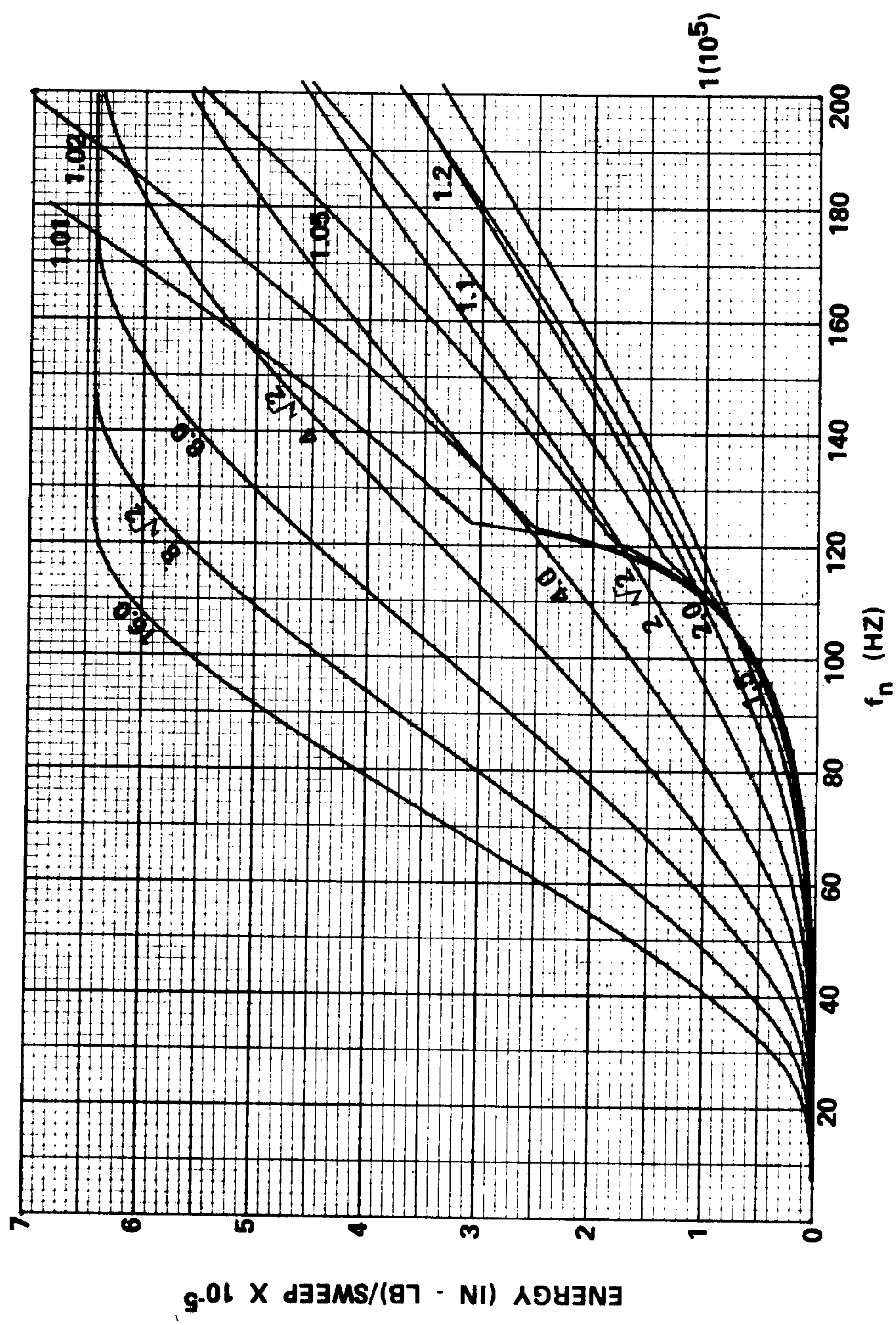

FIGURE 20 - E''_{S11} Versus fn for Family 2 d Values (Expanded)

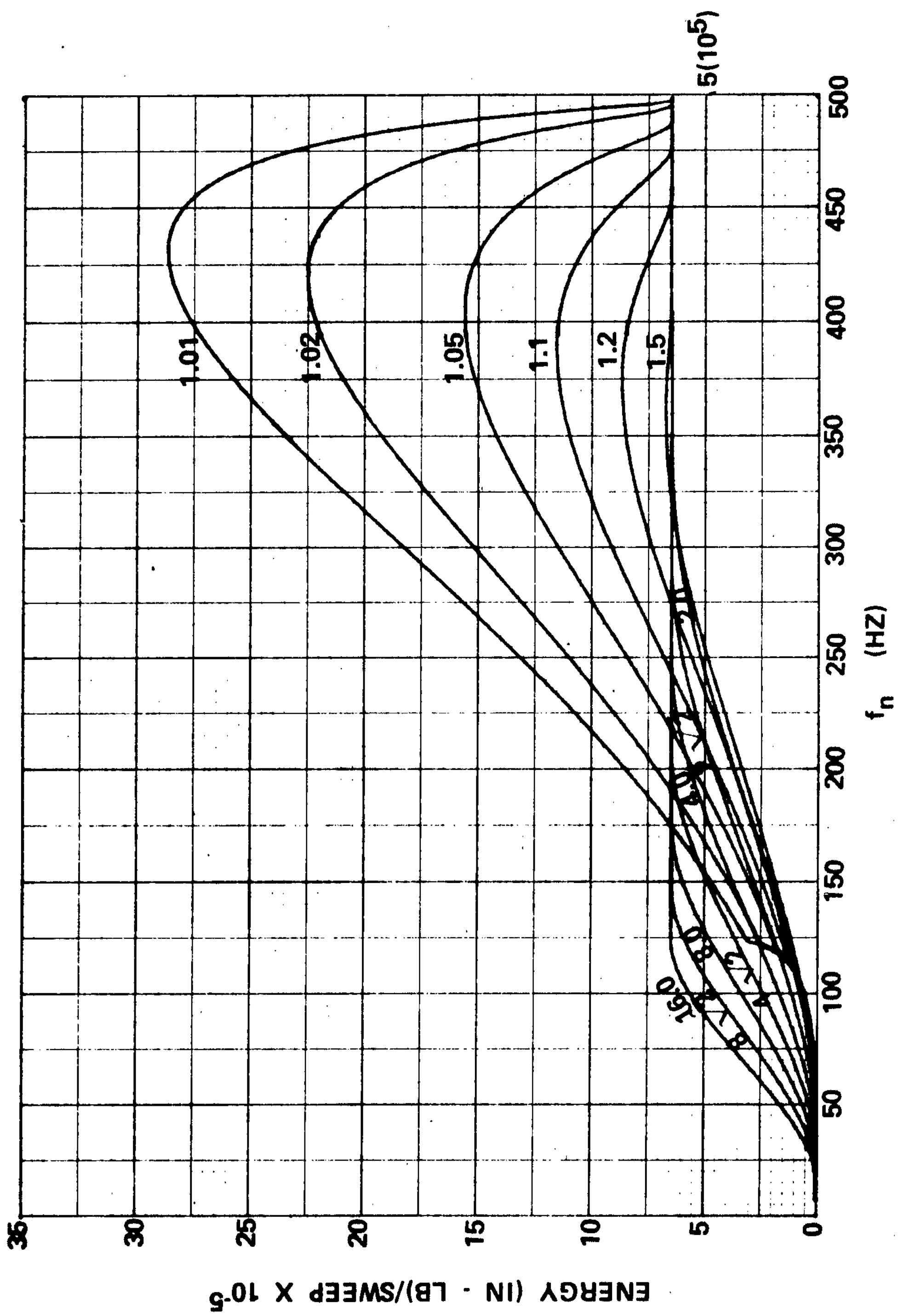

FIGURE 21 - E''_{S11} Versus fn for Family 2 d Values

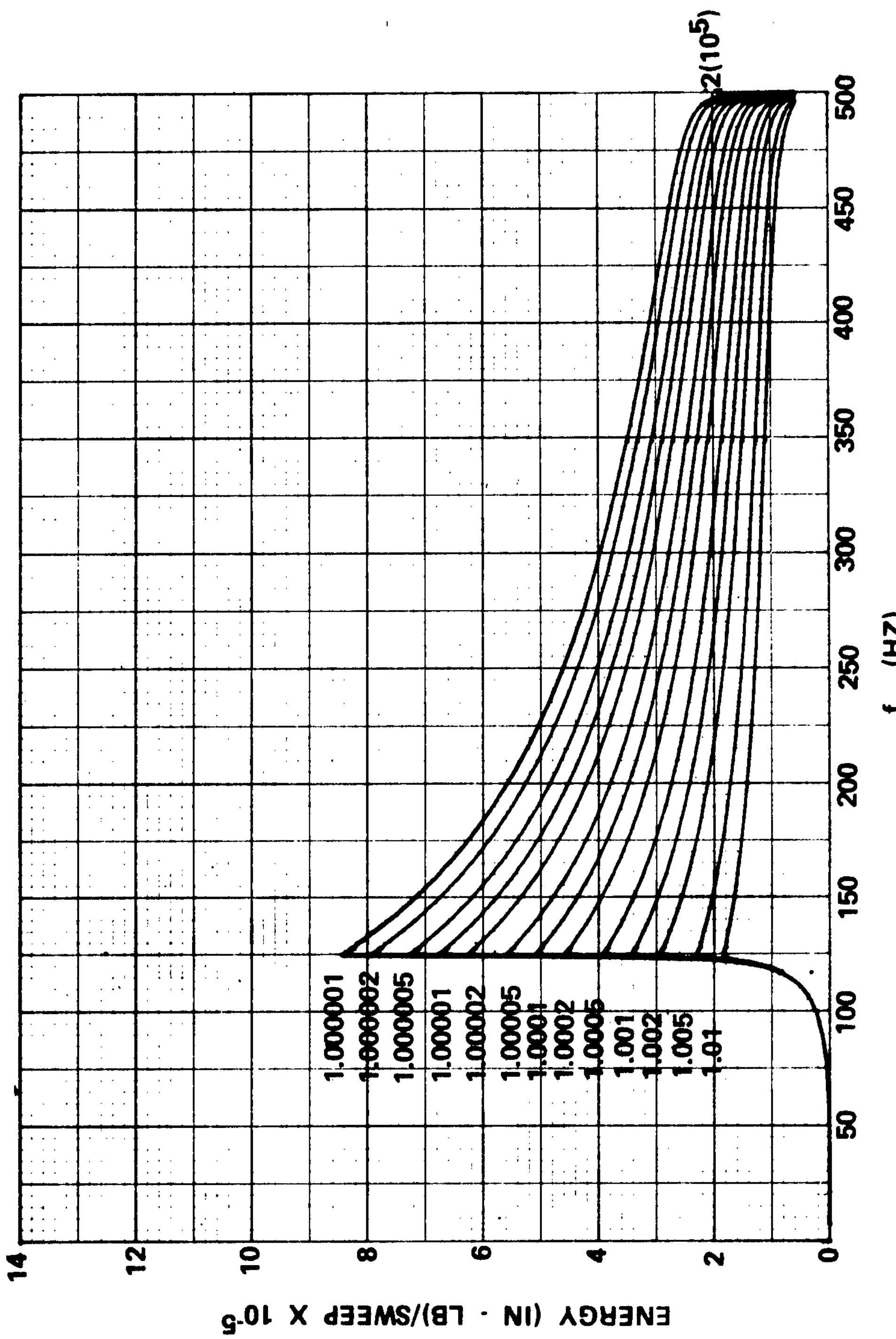

FIGURE 22 - E''_{S3} Versus fn for Family 1 d Values

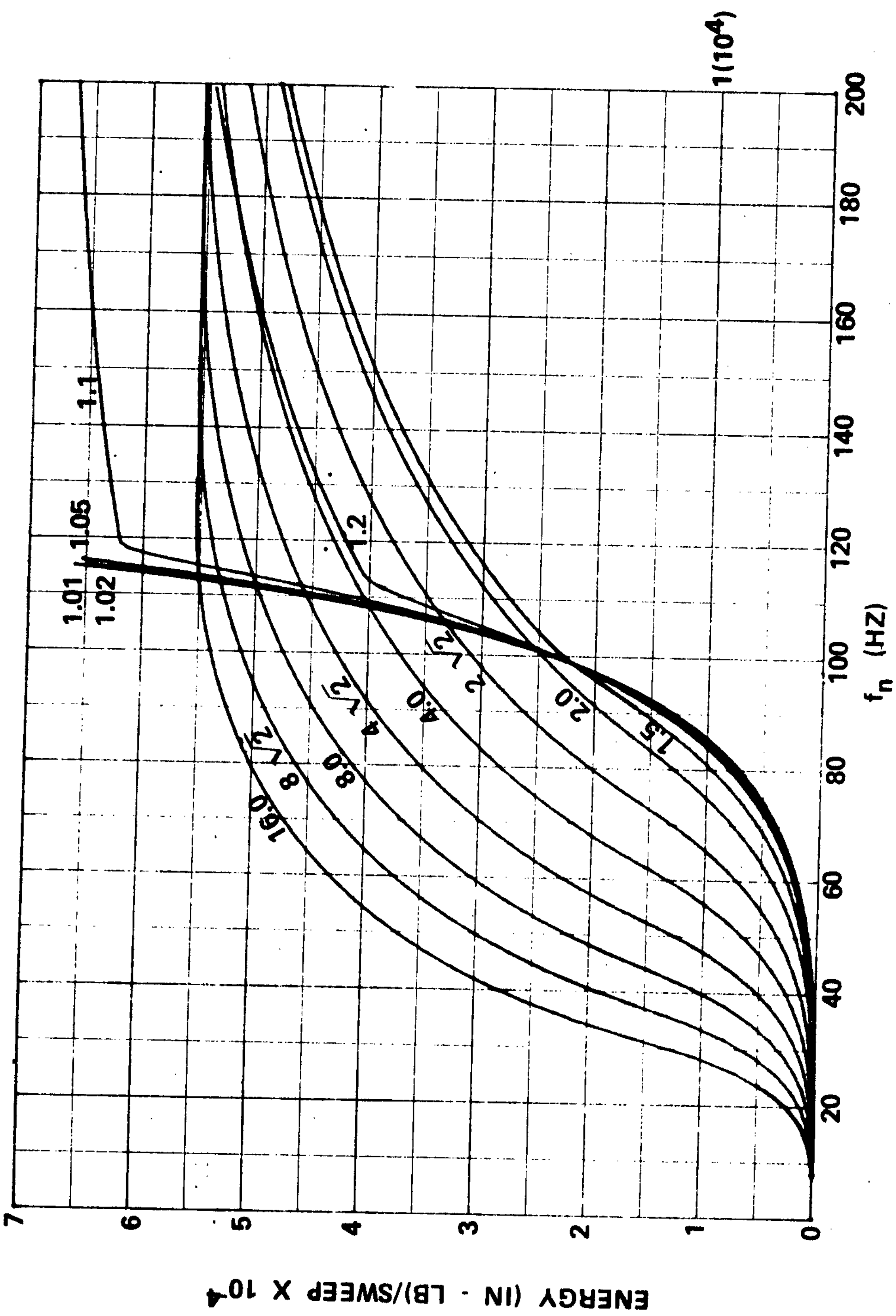

FIGURE 23 - E''_{S3} Versus fn for Family 2 d Values (Expanded)

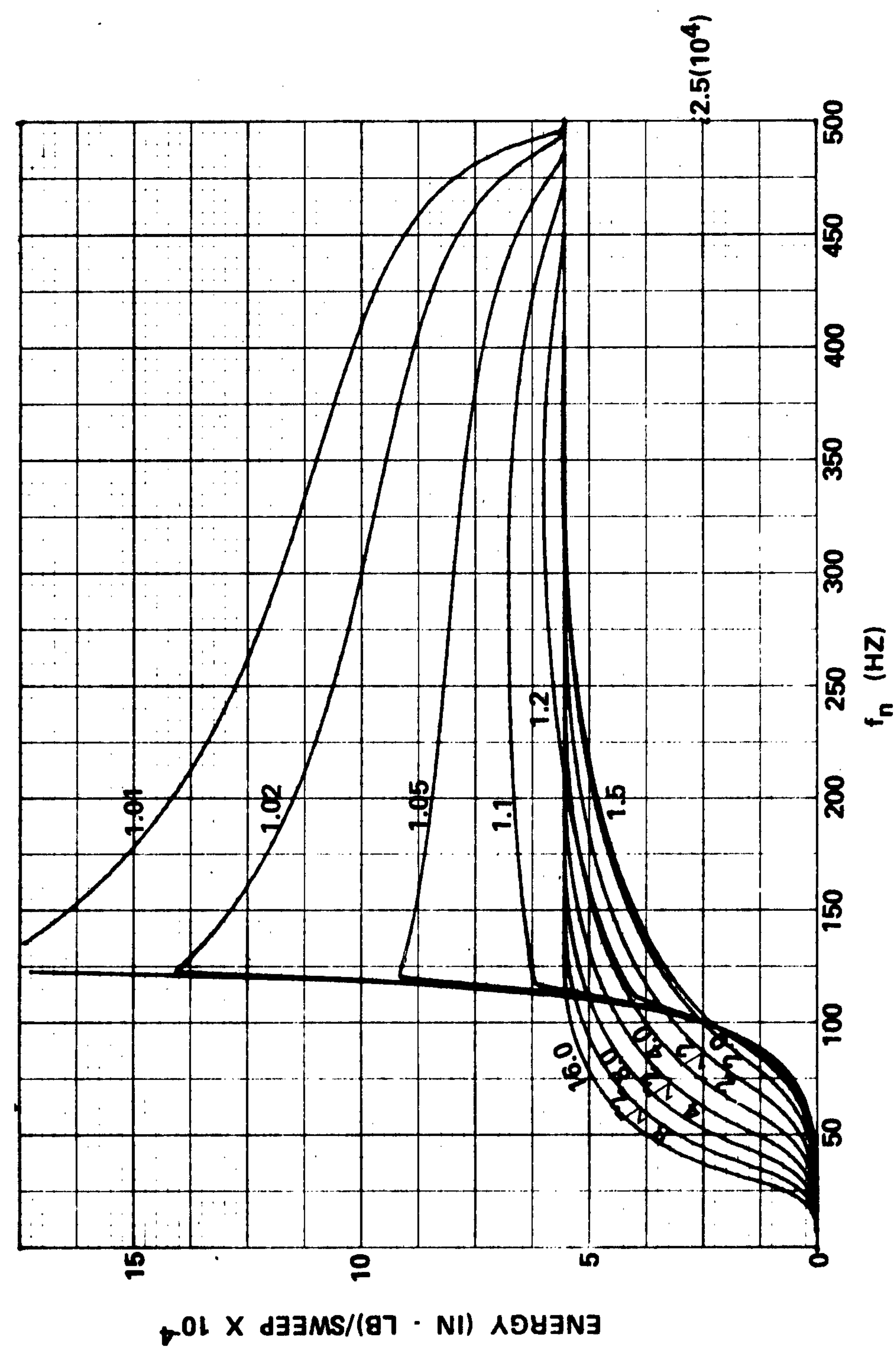

FIGURE 24 - E''_{S3} Versus fn for Family 2 d Values

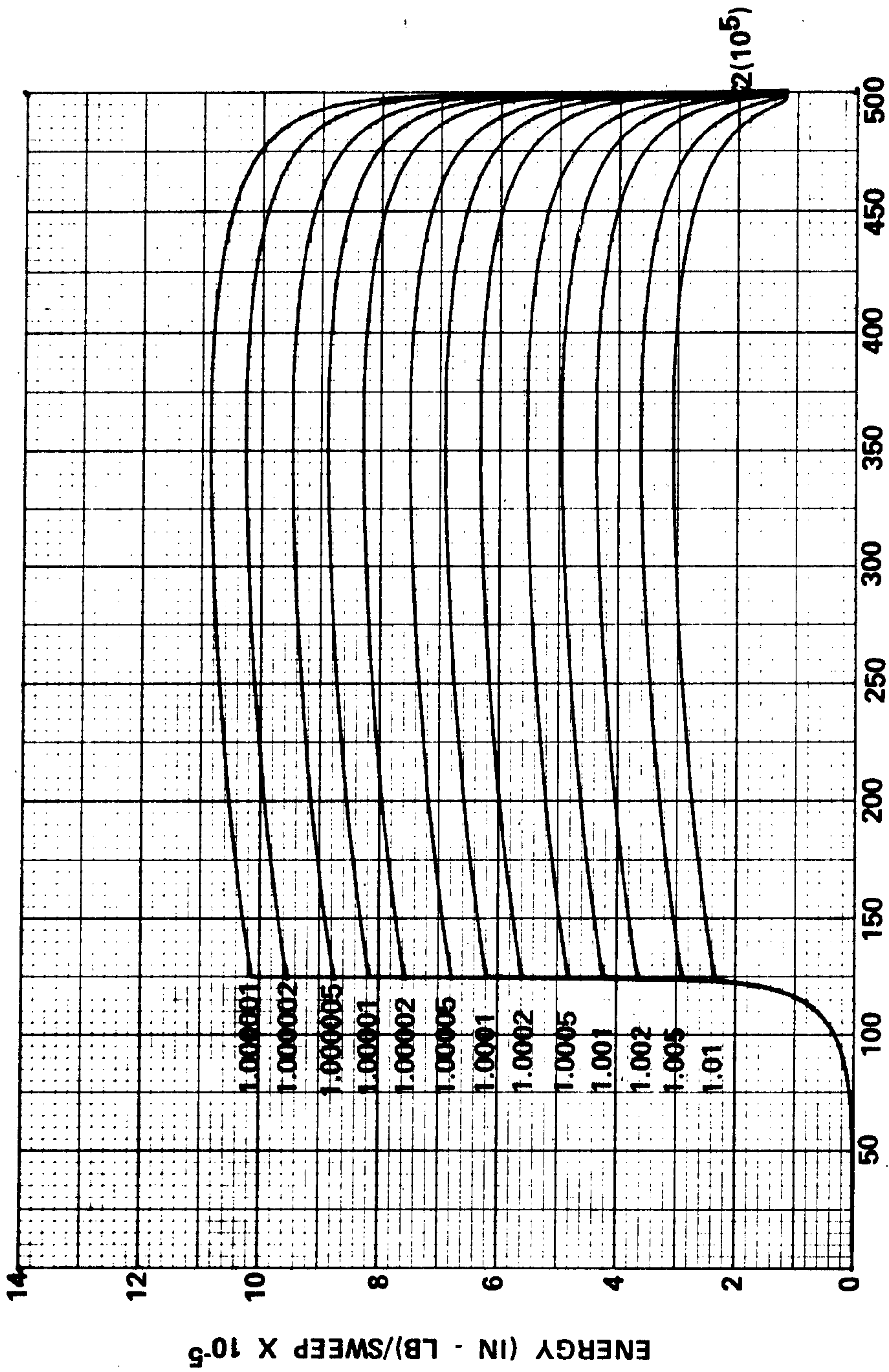

FIGURE 25 - E''_{S9} Versus fn for Family 1 d Values

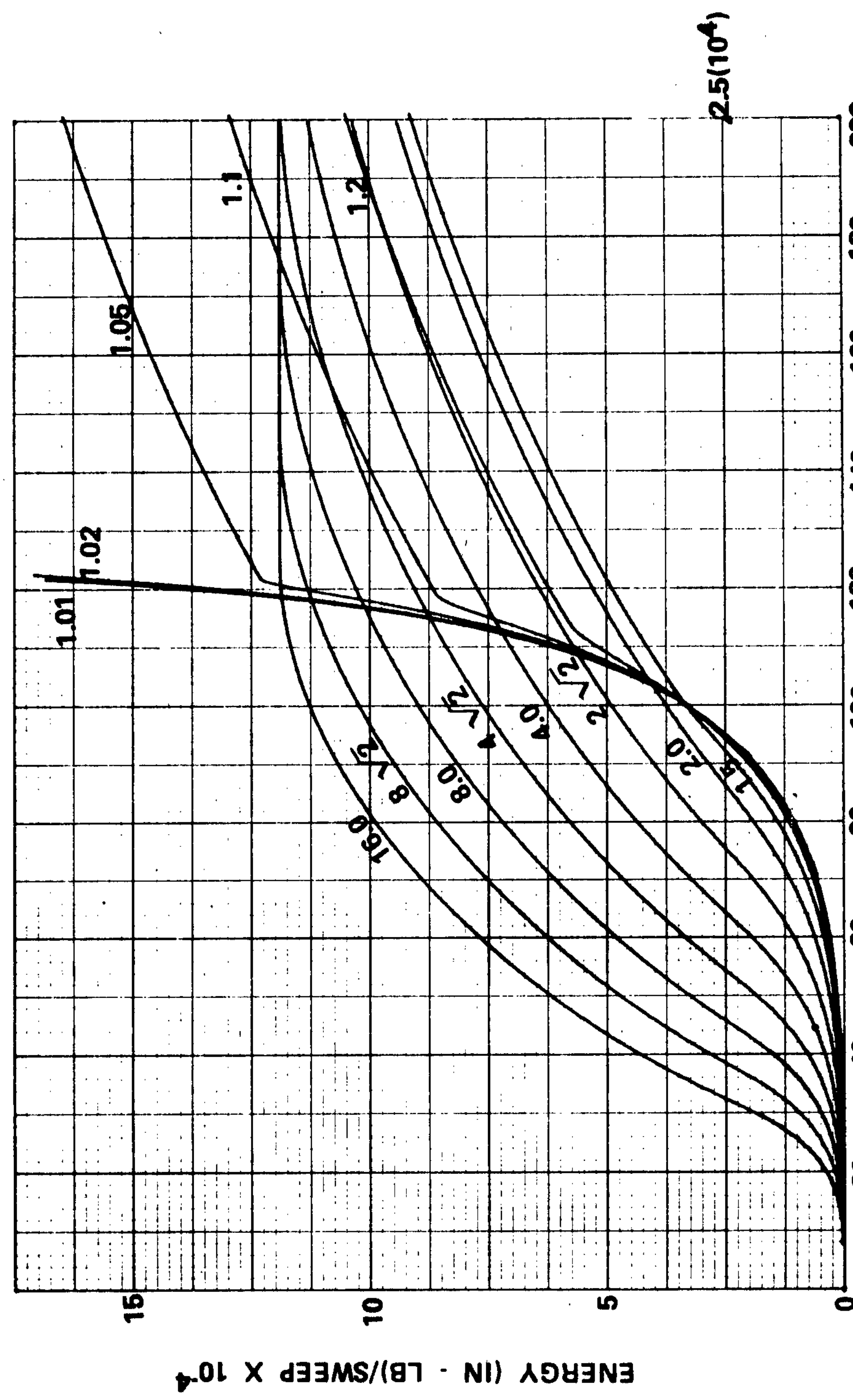

FIGURE 26 - E''_{S9} Versus fn for Family 2 d Values (Expanded)

FIGURE 27 - E''_{S9} Versus fn For Family 2 d Values

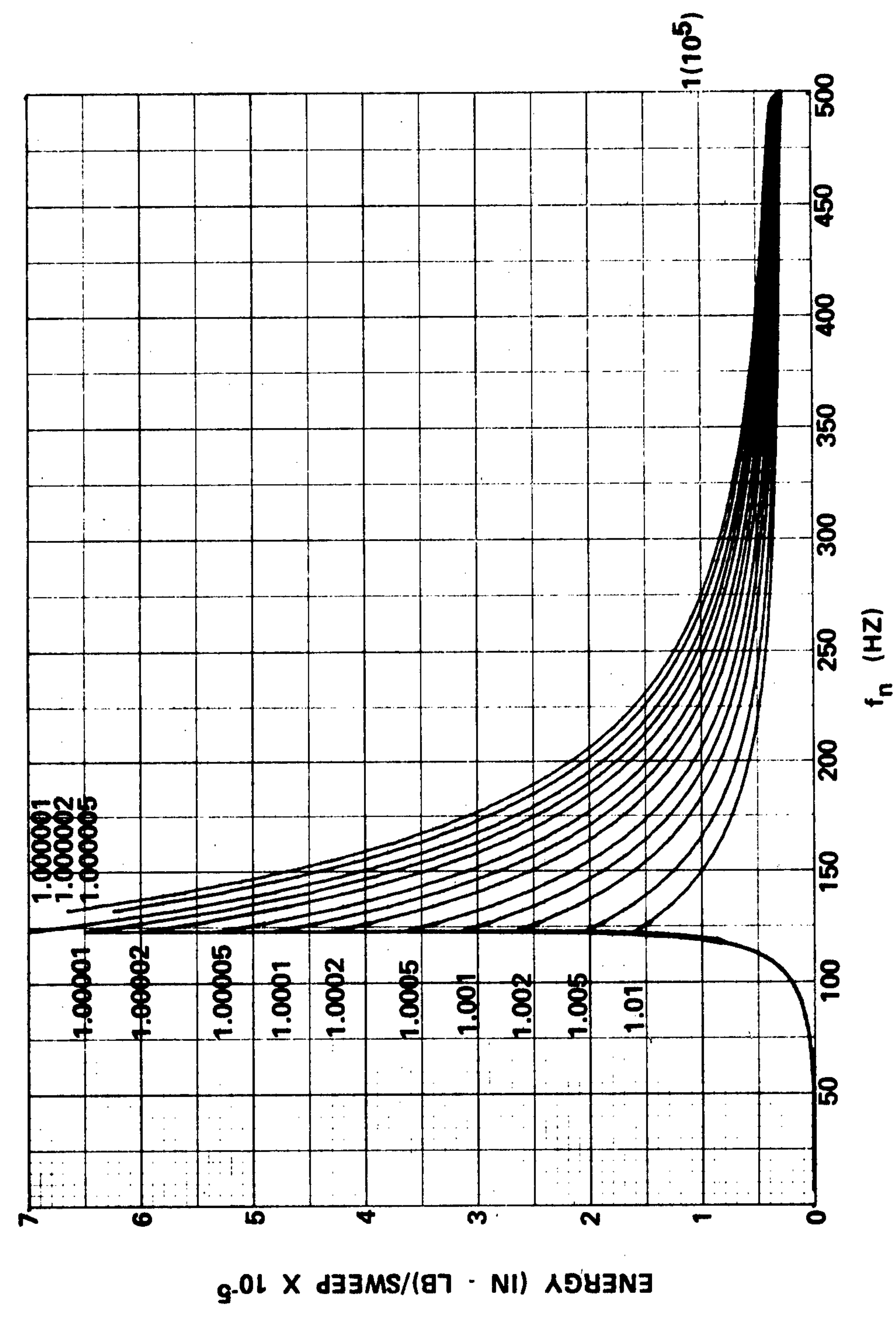

FIGURE 28 - E''_{S6} Versus fn for Family 1 d Values

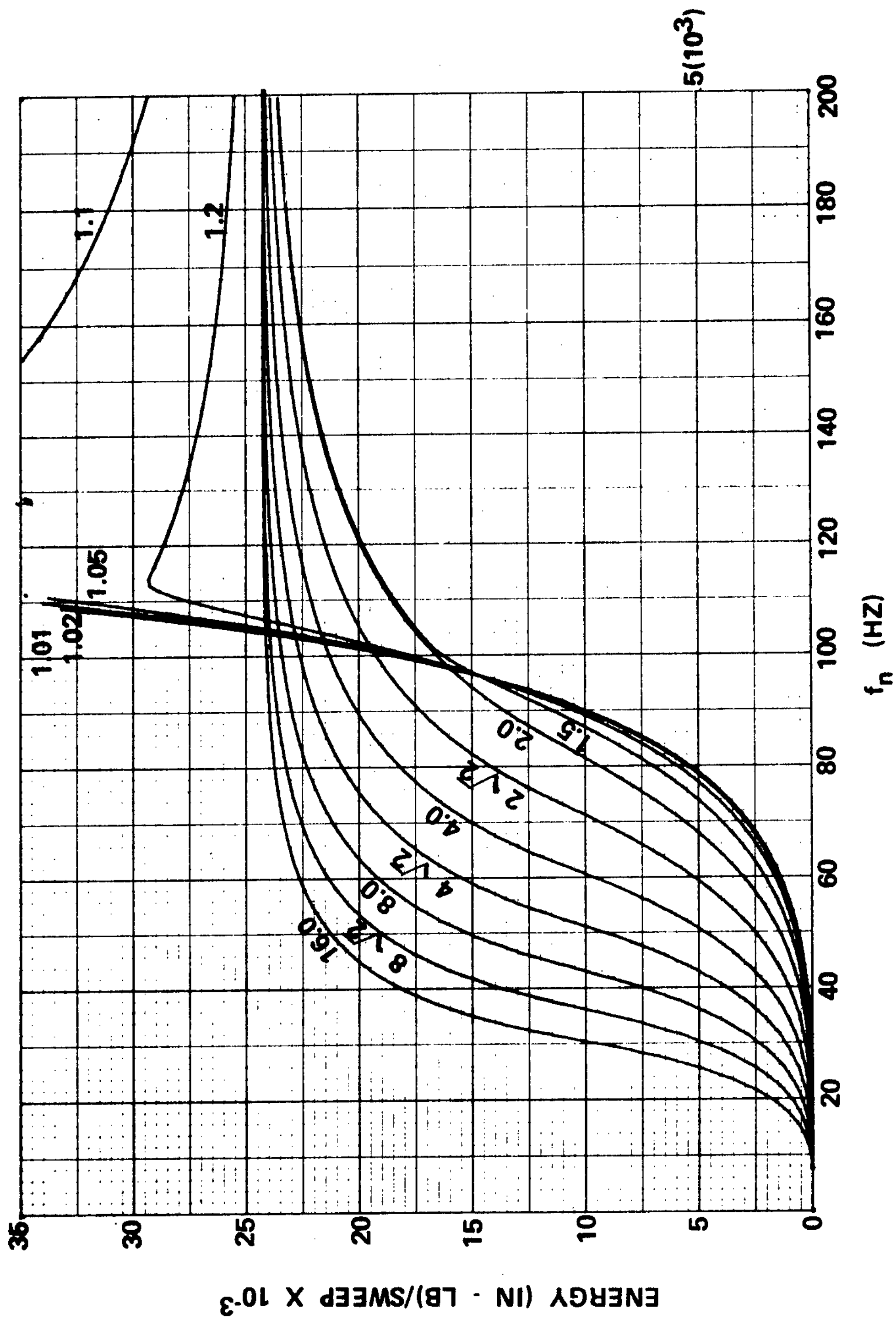

FIGURE 29 - E''_{S6} Versus fn for Family 2 d Values (Expanded)

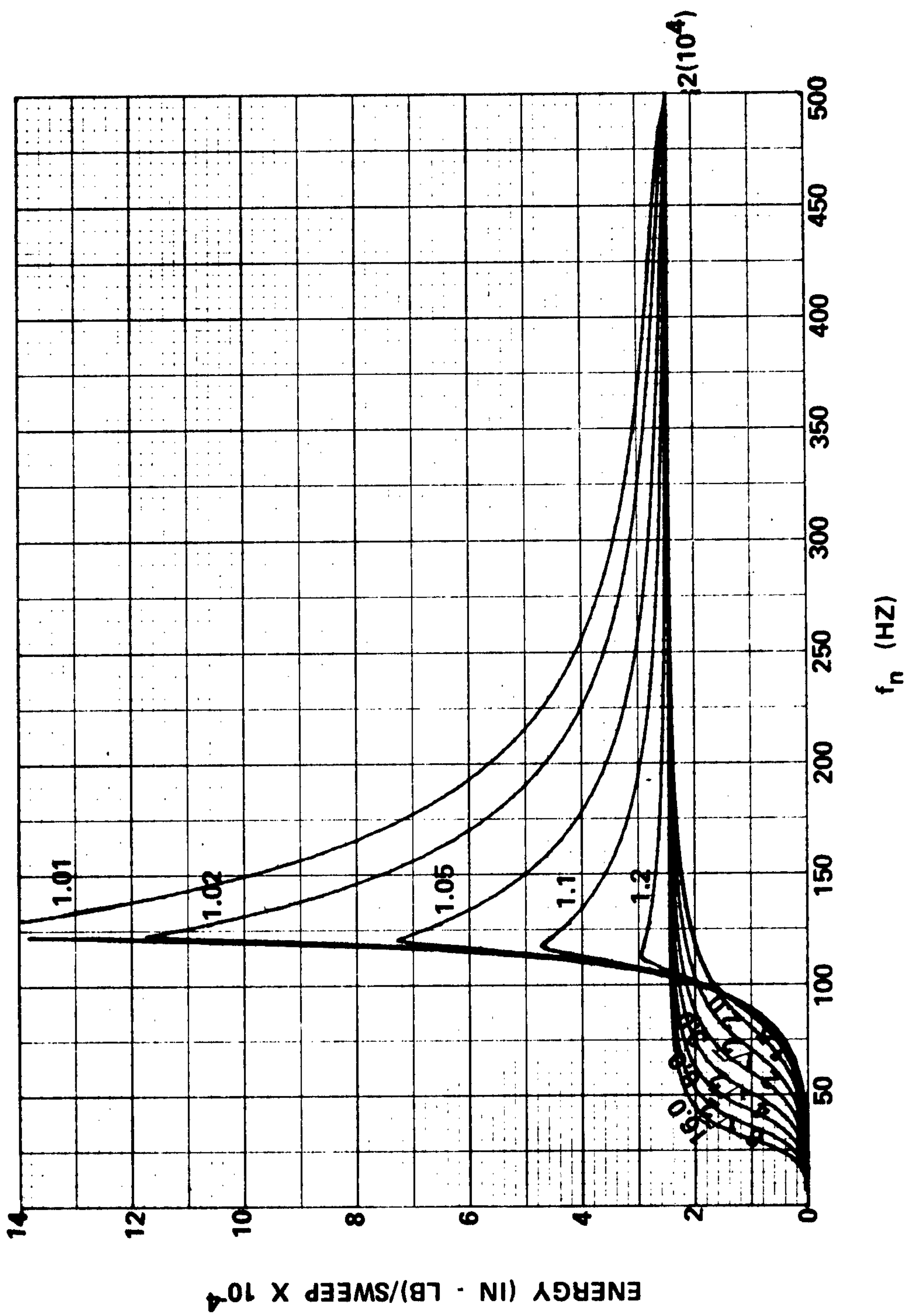

FIGURE 30 - E''_{S6} Versus fn for Family 2 d Values

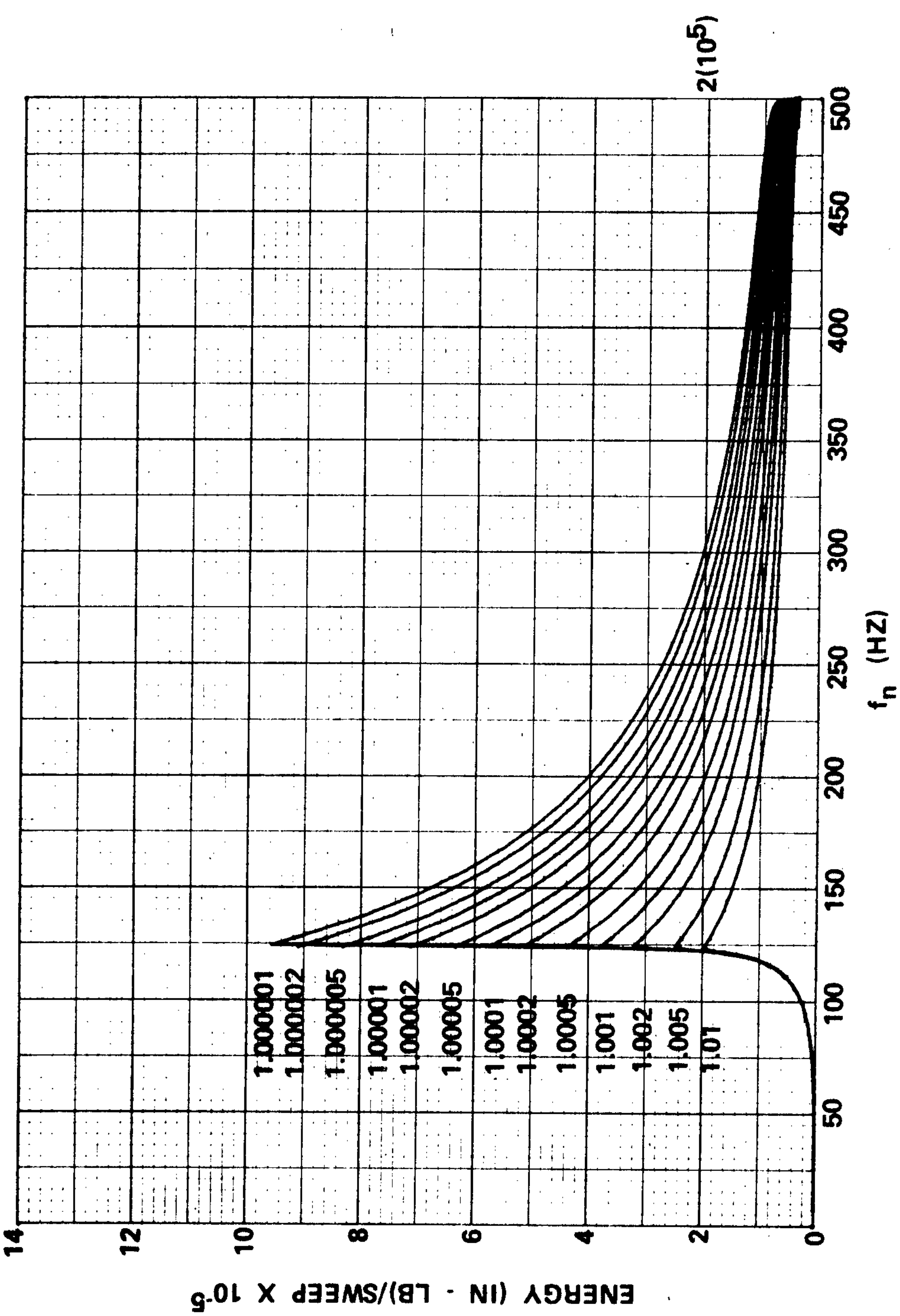

FIGURE 31 - E''_{S12} Versus fn for Family 1 d Values

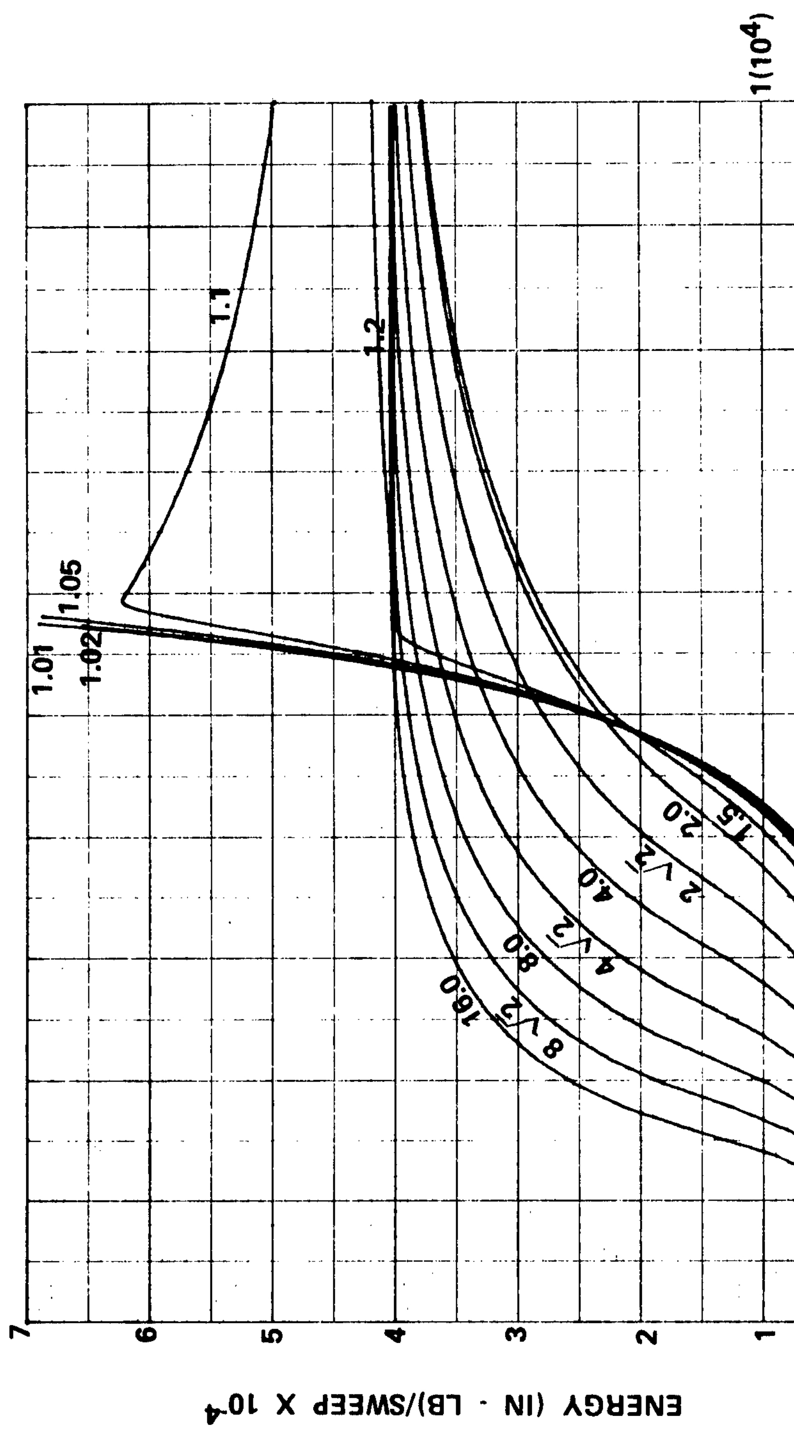

FIGURE 32 - E''_{S12} Versus fn for Family 2 d (Expanded)

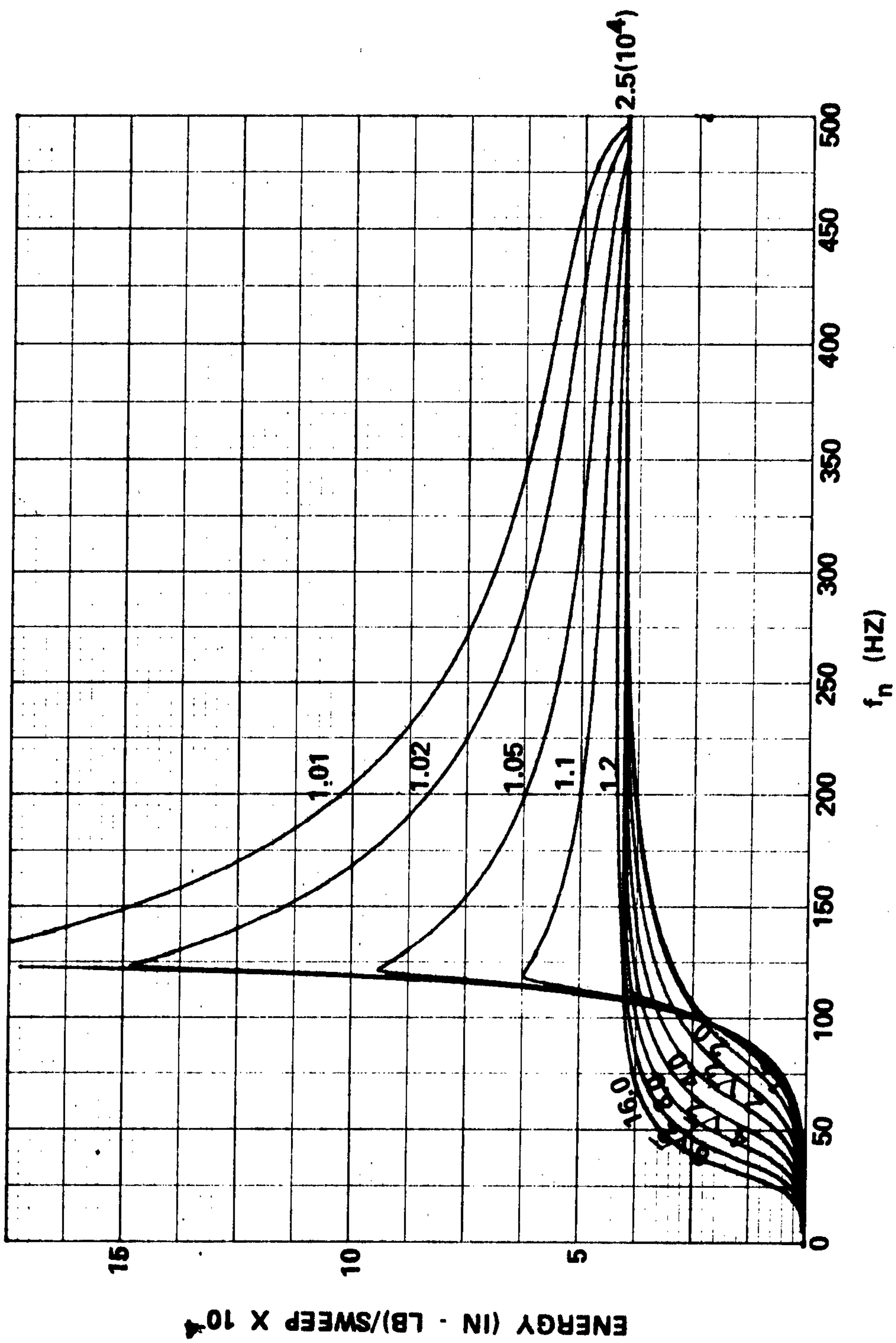

FIGURE 33 - E''_{S12} Versus fn for Family 2 d Values

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13. ABSTRACT

This report supplements the previous phase report covering response energy summation analysis for a rigidly-connected coulomb damped elastic structure model, when subjected to all controlled patterns of sinusoidal vibration. Specimen curve evaluations for extended parameter ranges are facilitated by the use of computer plotting programs.

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